What is Activity?
A Flavour of Discrete-event based Deepening

February 3, 2011

This note only presents a first introduction to the Activity concept. It aims at providing simple definitions to be further developed in simulation models.

1 Traditional qualitative definition in Simulation

Traditionally, in the simulation context, an accepted *qualitative definition of activity* is a phase of the system under study starting from an event and ending with another [1]. An event is also considered to cause a change in the state of a component. Information about the system is embedded in qualities indexed by strings ("burning", "waiting", etc.) We note an event ev_i by a couple (t_i, v_i), where t_i is the timestamp of the event, and v_i is the information associated to the event. Therefore, a *qualitative activity* A_j, indexed by j, is defined as: $A_j = (ev_i, ev_{i'})$. The set of activities consists of: $\alpha = \{A_j \mid j = 1, 2, 3...\}$.

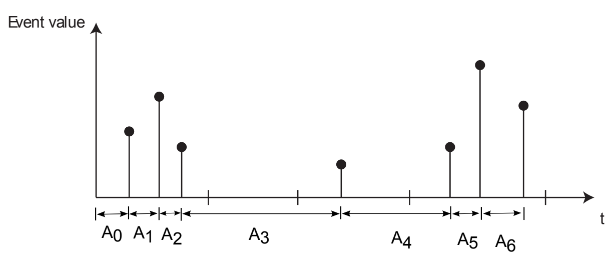

Figure 1: An example of qualitative activity definition.

2 A new quantitative richer definition?

An *original quantitative definition* of activity consists of considering activity as a measure of the number of events in an event set defined as $\xi = \{ev_i = (t_i, v_i) \mid i = 1, 2, 3, ...\}$. Formally, we define the event-based activity measure $\nu_H(t)$ as a function of time that provides the activity in a discrete event simulation, from t over a given time horizon H[2]:

$$\nu_H(t) = \frac{|\{ev_i = (t_i, v_i) \in \xi \mid t \leq t_i < t + H\}|}{H}$$

Activity is a measure of the event rate, or event frequency, in an event set. The qualitative differences of influence of events on the state of the dynamic system is voluntarily neglected here. Only the quantity of events over a period of time is taken into account. For example, assuming the event trajectory depicted in Figure 2, the activity of the system corresponds to the following values for different time horizons: $\nu_{10}(t) = 0.3$, $\nu_{20}(t) = 0.15$, $\nu_{30}(t) \simeq 0.133$, $\nu_{40}(t) = 0.175$.

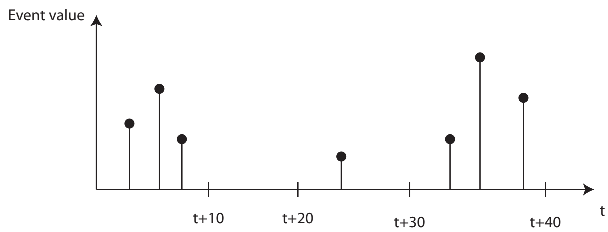

Figure 2: An example of event trajectory.

For the sake of simplicity, the activity measure is usually noted $\nu(t)$ (making implicit the dependency on the time horizon H).

3 Activity in components

A component corresponds to a Discrete Event System Specification, which is a tuple, denoted as $DEVS = < X, Y, S, \delta, \lambda, \tau >$, where X is the *set of input values*, Y is the *set of output values*, S is the *set of partial sequential states*, $\delta : Q \times (X \cup \{\emptyset\}) \to S$ is the *transition function*, where $Q = \{(s, e) \,|s \in S, 0 \leq e \leq \tau\,(s)\}$ is the *total state set*, e is the *time elapsed* since the last transition, is the *null input value*, $\lambda : S \to Y$ is the *output function*, $\tau : S \to \mathbb{R}^+_{0,\infty}$ is the *time advance function*.

If no event arrives at the system, it will remain in partial sequential state s for time $\tau\,(s)$. When $e = \tau\,(s)$, the system produces an output $\lambda\,(s)$, then it changes to state $(\delta(s, e, x), e) = (\delta(s, \tau\,(s)\,,), 0)$, which is defined as an *internal transition*. If an external event, $x \in X$, arrives when the system is in state (s, e), it will change to state $(\delta(s, \tau\,(s)\,, x), 0)$, which is defined as an *external transition*.

Denoting new sequential states as s', activity corresponds to:

- Activity A_{ext}, related to the counting, n_{ext}, of *external transitions* $\delta_{ext}(s, x) = (\delta(s, \tau\,(s)\,, x), 0)$, over a time period $[t, t']$:

$$\begin{cases} s' = \delta_{ext}(s,e,x) \Rightarrow n_{ext} = n_{ext} + 1 \\ A_{ext} = \frac{n_{ext}}{t'-t} \end{cases}$$

- Activity A_{int}, related to the counting, n_{int}, of *internal transitions* $\delta_{int}(s) = (\delta(s, \tau(s),), 0)$, over a time period $[t, t']$:

$$\begin{cases} s' = \delta_{int}(s,e) \Rightarrow n_{int} = n_{int} + 1 \\ A_{int} = \frac{n_{int}}{t'-t} \end{cases}$$

- Total activity is equal to:

$$A = A_{ext} + A_{int}$$

4 Activity in Space

The activity measure is used to determine the sub-regions of the Cartesian coordinate space [2] through:

- Activity region in space:

$$\mathcal{AR}^{\mathcal{P}}(t) = \{p \in \mathcal{P} \mid \nu^p(t) > 0\}$$

- Inactivity region in space:

$$\overline{\mathcal{AR}^{\mathcal{P}}(t)} = \{p \in \mathcal{P} \mid \nu^p(t) = 0\}$$

We consider now the function of reachable states in time and space as $q : \mathcal{P} \times \mathcal{T} \rightarrow Q$. We can define now the set of all reachable states in the state set Q, through time and space, through the *universe* $\mathcal{U} = \{q(p,t) \subseteq Q \mid p \in \mathcal{P}, t \in \mathcal{T}\}$.

Considering that all reachable states in time and space can be active or inactive, an activity-based partitioning of $\mathcal{P}$ can be achieved: $\forall t \in \mathcal{T}, \mathcal{P} = \mathcal{AR}^{\mathcal{P}}(t) \cup \overline{\mathcal{AR}^{\mathcal{P}}(t)}$.

Figure 3 depicts activity values for two-dimensional Cartesian coordinates $X \times Y$. This is a neutral example, which can represent whatever activity measures in a Cartesian space (fire spread, brain activity, etc.)

5 Activity regions in Cartesian coordinates for composite models

In spatialized models[1] components are localized into a Cartesian coordinate space $\mathcal{P}$. Each component c is assigned to a position $c_p \in \mathcal{P}$. Applying the definition of activity regions in space to components, we obtain:

[1] A model is said to be *spatialized* when the phenomenon under study has a spatial extension. This requires that states have a richer structure than just scalar values to cope with the discretization of a spatially embedded phenomenon. Examples of spatialized models include cellular automata and L-systems.

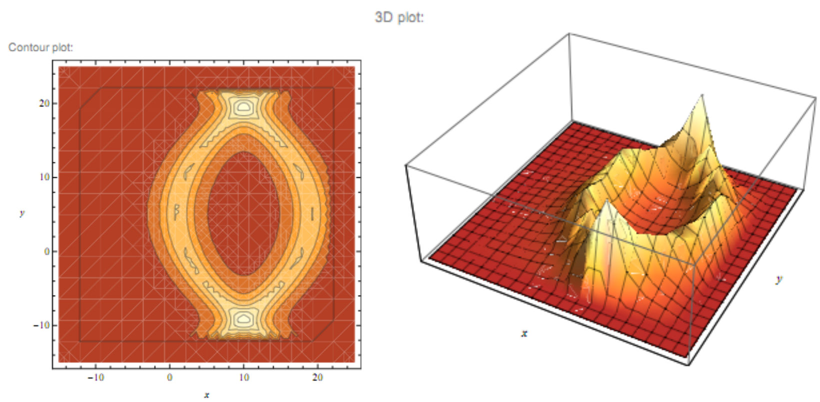

Figure 3: 2D and 3D visualization of activity level in a 2D space. x and y represent Cartesian coordinates. The activity amplitude (real value), of each coordinate, is represented in the third dimension.

$$\mathcal{AR}^{\mathcal{C}}(t) = \left\{ c \in \mathcal{C} \mid c_p \in \mathcal{AR}^{\mathcal{P}}(t) \right\}$$

$\mathcal{AR}^{\mathcal{P}}(t)$ specifies the coordinates where activity occurs. Consequently, active components correspond to the components localized at positions p.

6 Open Research

The quantification of activity *in* and *of* components opens new theoretical directions, *e.g.*, in:

- Machine Learning, where the activity as a *usage* of components in the search space can be correlated to their score.
- Networks, where activity provides an indication of the frequency of node accesses as well as indexes for *information paths*.
- Automatic Modeling and Simulation, where the combination of the use of activity in Machine Learning and Networks can be used to build automatically simulation models.
- ...

In relation to these theoretical directions, application domains are also large:

- In neurosciences, through the mapping between the activity of components/networks and neurons/brainRegions,

- In ecology, through the analogy between activity and the energy used by organisms to survive and evolve,
- In economics, through the comparison of decision paths, characterized through their activity.
- In propagation processes, activatabiliy and activity can be used at run-time for optimization, for activatability pre-processing (*e.g.*, in fire spread, where the vegetation is expected to burn, etc.)
- ...

Notice that application domains are orthogonal to theoretical directions (*e.g.*, simulation models of decisions in neuroscience can be developed together with economic ones, simulation models of propagations in a brain can be developed together with fire spreading ones, etc.)

References

[1] Osman Balci. The implementation of four conceptual frameworks for simulation modeling in high-level languages. In *WSC '88: Proceedings of the 20th conference on Winter simulation*, pages 287–295, New York, NY, USA, 1988. ACM.

[2] Alexandre Muzy, Luc Touraille, Hans Vangheluwe, Olivier Michel, Mamadou Kaba Traoré, and David R. C. Hill. Activity regions for the specication of discrete event systems. In *Spring Simulation Multi-Conference Symposium On Theory of Modeling and Simulation (DEVS)*, pages 176–182, 2010.