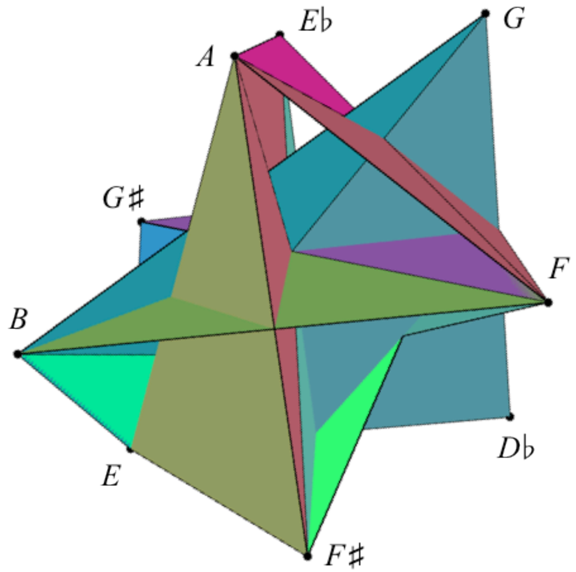
Musical Symbolic Representations and Spatial Computing



Louis BIGO

Thesis defense

December 13th 2013

LACL – LCP team, Université Paris-Est Créteil

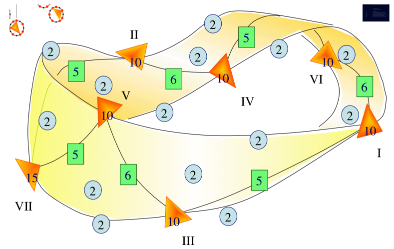
IRCAM – Musical Representations team

lacl

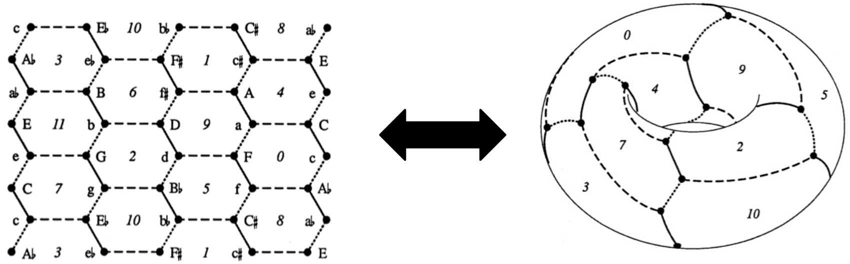
UNIVERSITÉ PARIS-EST

ircam Centre Pompidou

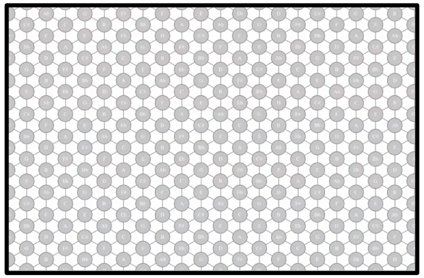
Space in Music Theory



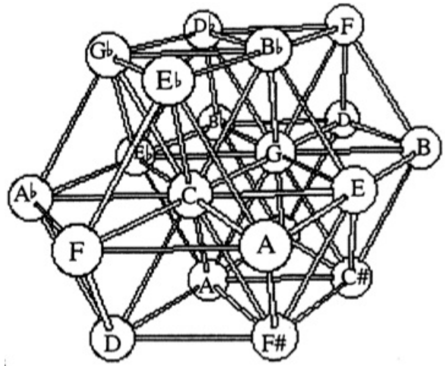
Tonality strip [Mazzola]



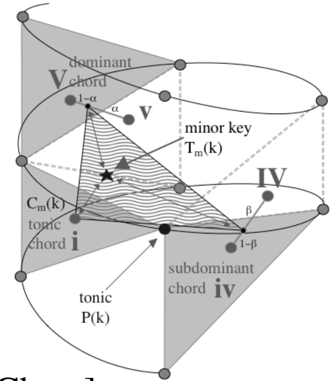
Chicken Wire Torus [Douthett & Steinbach]



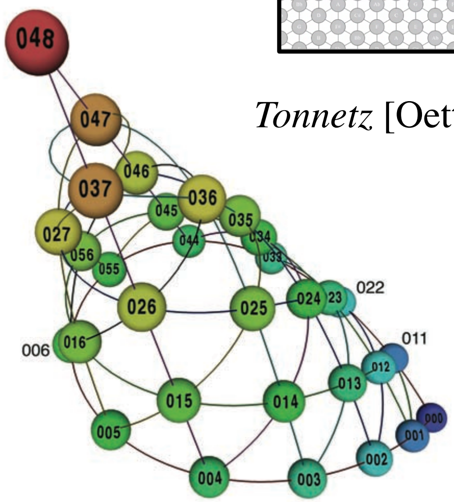
Tonnetz [Oettingen, Riemann]



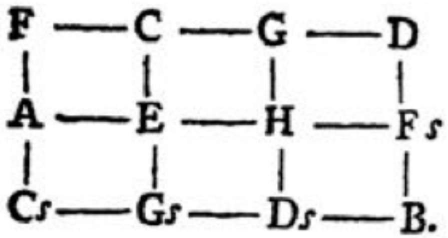
3D *Tonnetz* [Gollin]



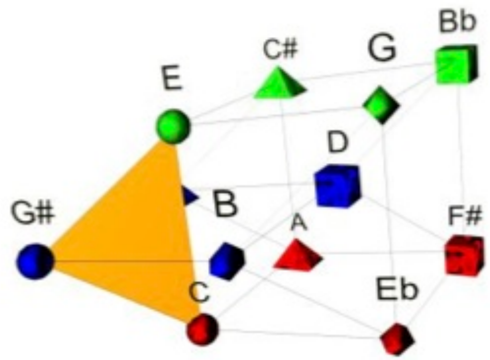
Spiral Array [Chew]



Orbifolds [Tymoczko]



Speculum Musicum [Euler]



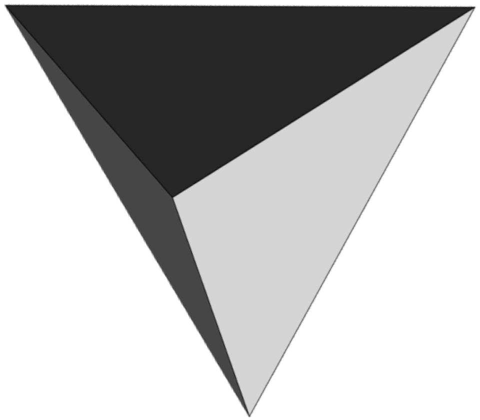
Model Planet [Barouin]

Space in Computer Science

- *Spatial Computing*

  Recent domain of computer science (≈ 2005)

- Importance of space in computation
  - Construction of a space
  - Motion in a space



- Different dedicated tools (MGS, etc.)

Outline

Bridging the gap between spatial computing and music theory

1. Proof of concept: a spatial study of all-interval series
2. Building chord spaces for music theory and analysis
3. Linking spaces for music generation and analysis
4. Conclusion and perspectives

Outline

Bridging the gap between
spatial computing and music theory

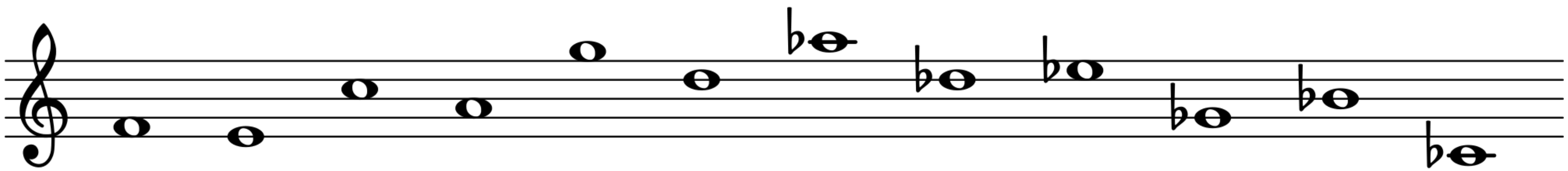
1. Proof of concept: a spatial study of all-interval series
2. Building chord spaces for music theory and analysis
3. Linking spaces for music generation and analysis
4. Conclusion and perspectives

All-Interval Series

- Motivation: enumeration of All-Interval Series (AIS)
- AIS: sequence of 12 notes including
  - The 12 pitch classes
  - The 11 interval classes
  - Example: Alban Berg's *Lyric Suite*

© Tom Miller

A. Berg (1885 – 1935)



Notes: *F* *E* *C* *A* *G* *D* *Ab* *Db* *Eb* *Gb* *Bb* *Cb*

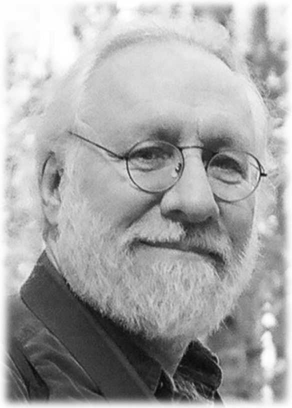
Intervals: *M7* *m6* *M6* *m7* *P5* *TT* *P4* *M2* *m3* *M3* *m2*

- 46272 AIS (1928 normalized AIS [Riotte62])

All-Interval Series

Fortran code

```
      DIMENSION N(12),I(12), NX(11),IX(11)
      DATA J,K,N/1,12*0,6/,I,NX/6,22*0/,IX/11*0/
C MOVE RIGHT
7     J=J+1
      IF(J.GT.11)GO TO 1
      N(J)=1
C IS N(J) A DUPLICATED NOTE?
4     IF(NX(N(J)).EQ.0)GO TO 2
5     N(J)=N(J)+1
      IF(N(J).EQ.6)GO TO 5
      IF(N(J).GT.11)GO TO 3
      GO TO 4
C CALCULATE I(J), THE INTERVAL
2     I(J)=N(J)-N(J-1)
      IF(I(J).LT.0)I(J)=I(J)+12
C IS I(J) A DUPLICATED INTERVAL?
6     IF(IX(I(J)).EQ.1)GO TO 5
      NX(N(J))=1
      IX(I(J))=1
      GO TO 7
C CALCULATE THE 11TH INTERVAL
1     I(J)=N(12)-N(11)
      IF(I(J).LT.0)I(J)=I(J)+12
      IF(IX(I(J)).EQ.1)GO TO 3
C LAND HERE WHEN AN AIS IS FOUND
      K=K+1
C STATEMENT BELOW IS OPTIONAL—SHORTENS THE TABLE
      IF(K.GE.1929)STOP
      WRITE(6,8)K,N,I
8     FORMAT(I5,2(4X,12I3))
C MOVE LEFT
3     J=J-1
      IF(J.EQ.1)STOP
      NX(N(J))=0
      IX(I(J))=0
      GO TO 5
      END
```



R. Morris

[Morris, Star – 1974]

Spatial Interpretation of AIS

- AIS as *movement* in some *space*
- What kind of *space*?
  - Search space

    Spatialization of pitch and interval classes
- What kind of *movement*?
  - Subspace with structural properties

    Uniqueness of pitch and interval classes

Spatial Interpretation of AIS

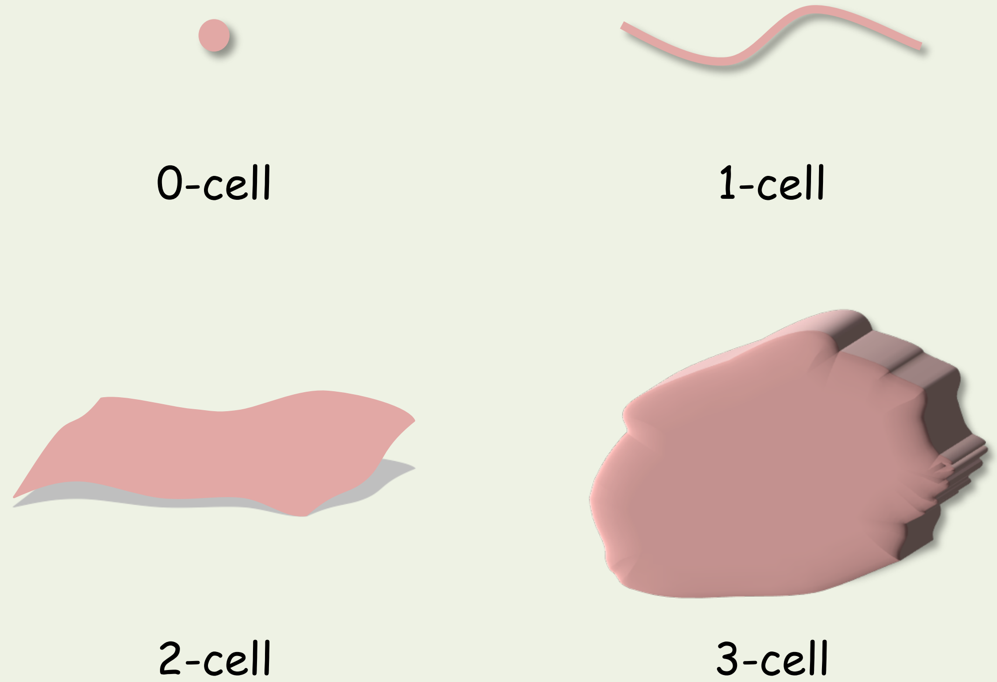
Toolbox: *Cellular complex*

- Partially ordered set $\mathcal{K} = (S, \leq)$

Spatial Interpretation of AIS

Toolbox: *Cellular complex*

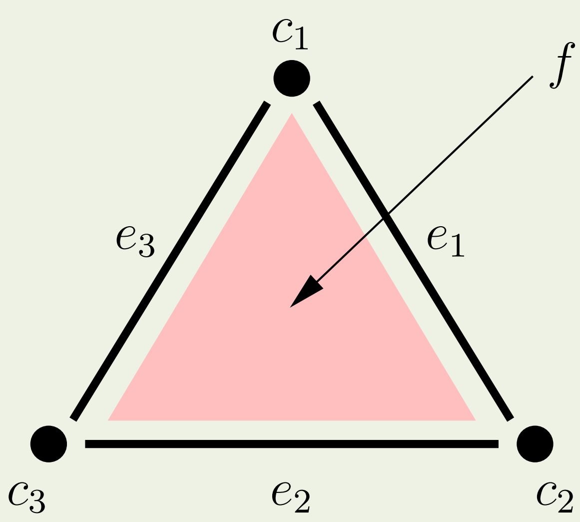
- Partially ordered set $\mathcal{K} = (S, \leq)$
- S : Collection of *topological cells*

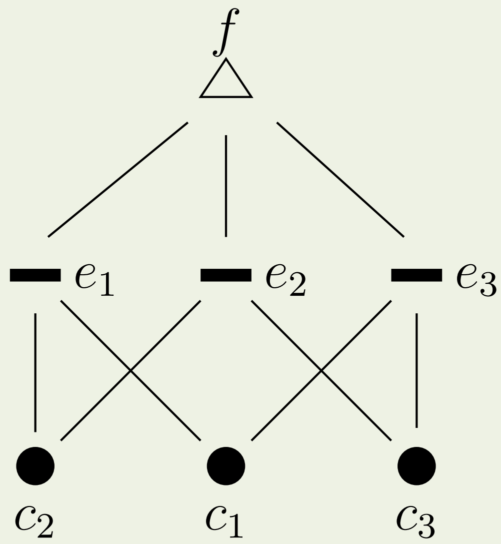


Spatial Interpretation of AIS

Toolbox: *Cellular complex*

- Partially ordered set $\mathcal{K} = (S, \leq)$
- S : Collection of *topological cells*
- $\leq$: *Incidence relationship*



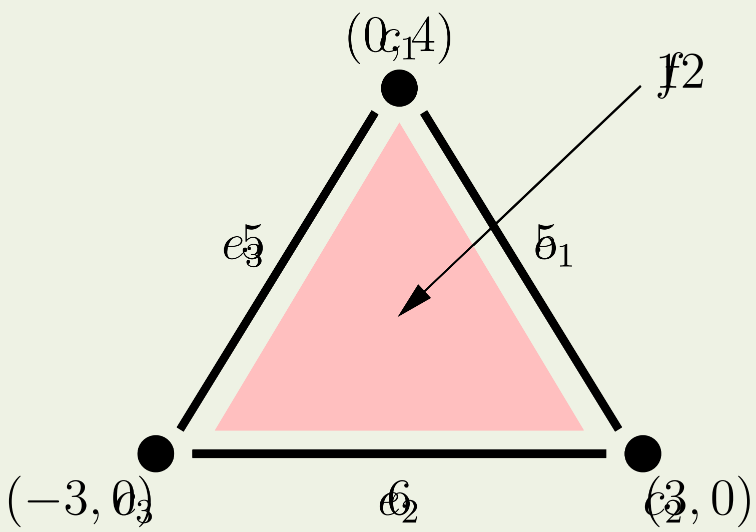


Spatial Interpretation of AIS

Toolbox: *Topological collection*

- *Labeled cellular complex*

  Partial function $C : \mathcal{K}$ -> V, where V is the set of labels



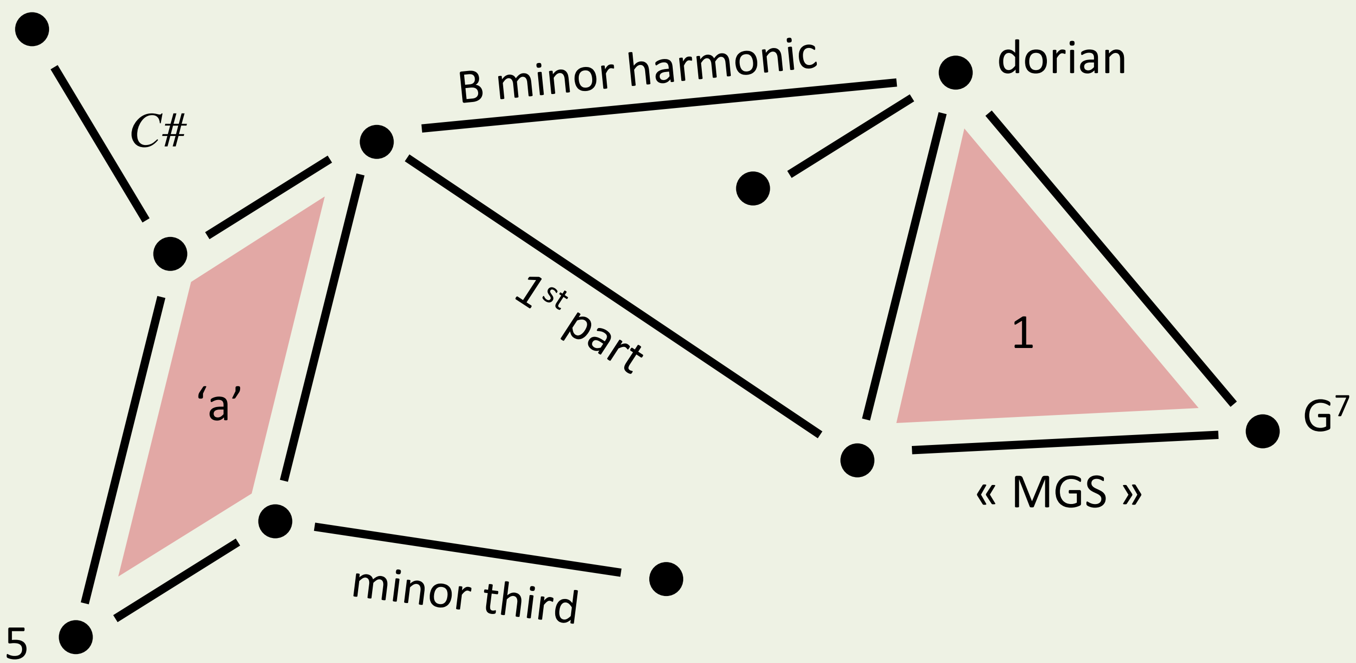
Spatial Interpretation of AIS

Toolbox: *Topological collection*

- *Labeled cellular complex*

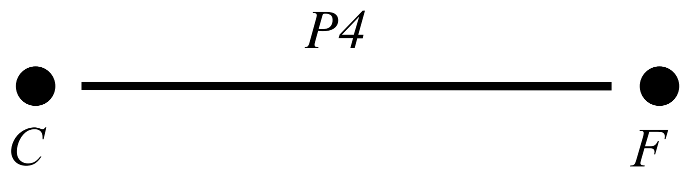
  Partial function $C : \mathcal{K} \rightarrow V$, where V is the set of labels

- Topological point of view of data structures (MGS)



Spatial Interpretation of AIS

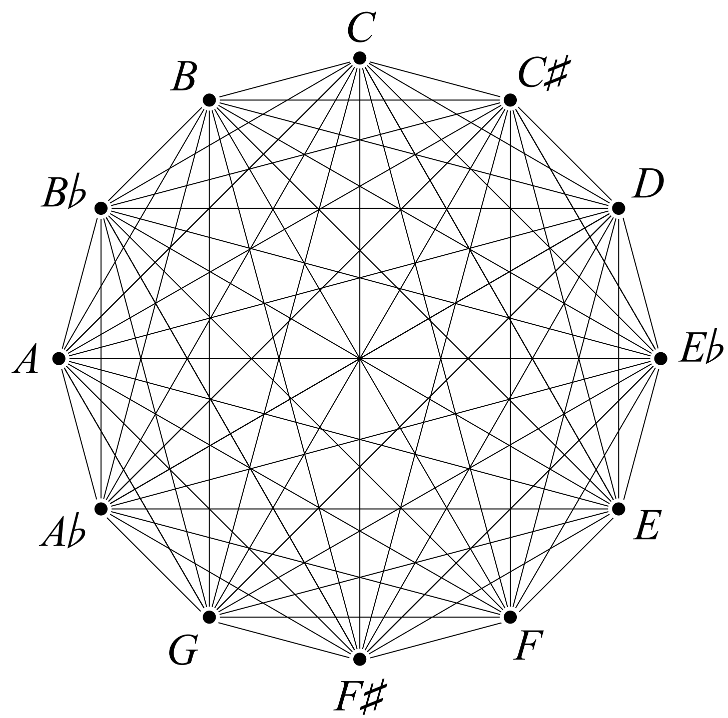
- AIS as *movement* in some *space*
- What kind of *space*?
  - Search space
    - Pitch classes: 0-cells
    - Intervals: 1-cells



- What kind of *movement*?
  - Subspace with structural properties

Spatial Interpretation of AIS

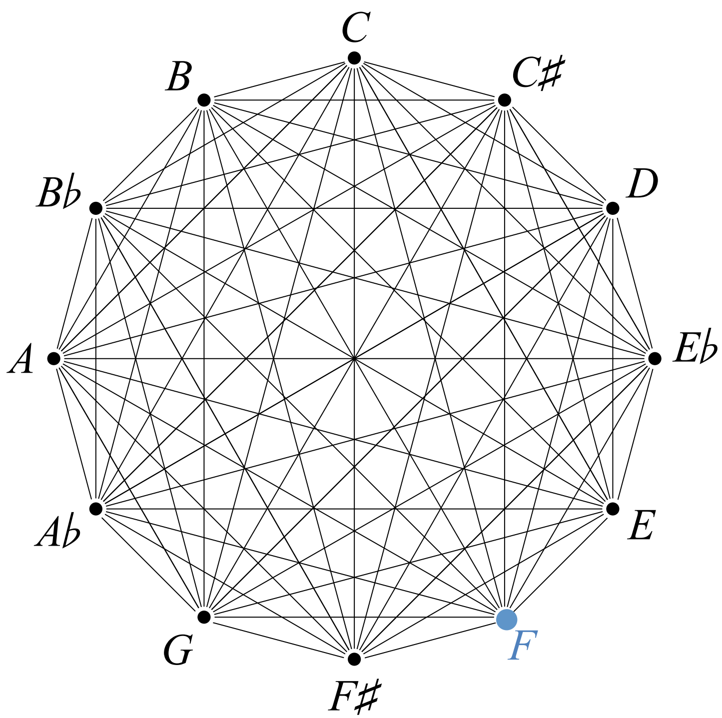
- AIS as *movement* in some *space*
- What kind of *space*?
  - Search space
    - Pitch classes: 0-cells
    - Intervals: 1-cells
- What kind of *movement*?
  - Subspace with structural properties



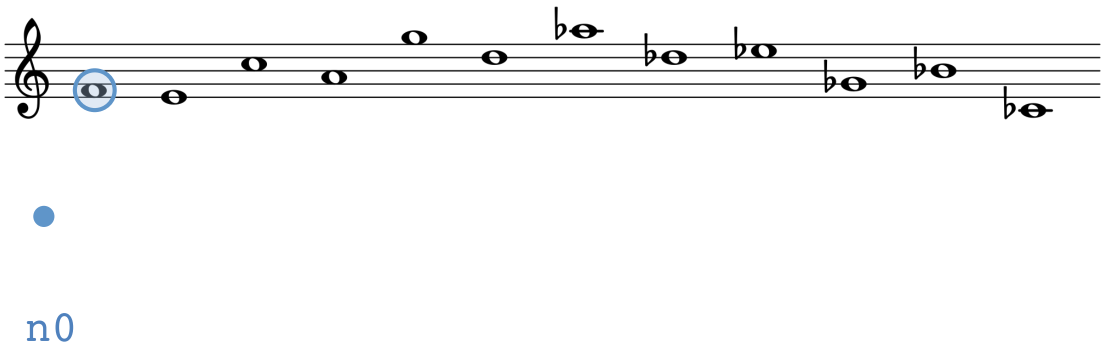
Complete graph K_{12}

Spatial Interpretation of AIS

- AIS as *movement* in some *space*
- What kind of *space*?
  - Search space
    - Pitch classes: 0-cells
    - Intervals: 1-cells
- What kind of *movement*?

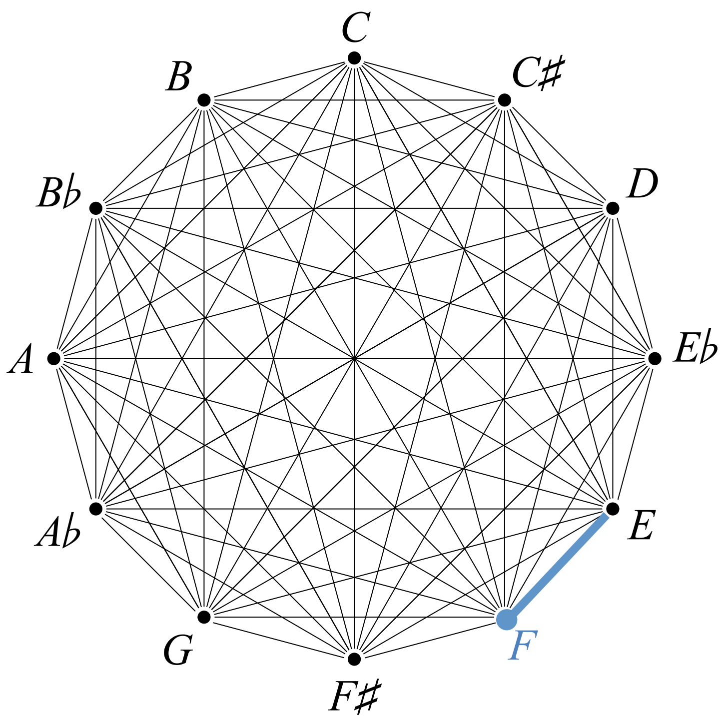


Complete graph K_{12}

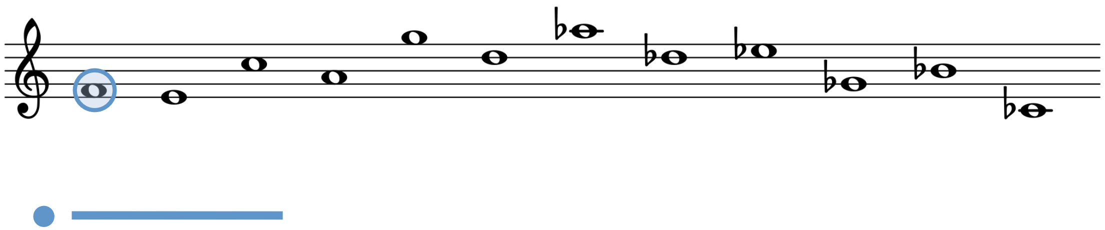


Spatial Interpretation of AIS

- AIS as *movement* in some *space*
- What kind of *space*?
  - Search space
    - Pitch classes: 0-cells
    - Intervals: 1-cells
- What kind of *movement*?



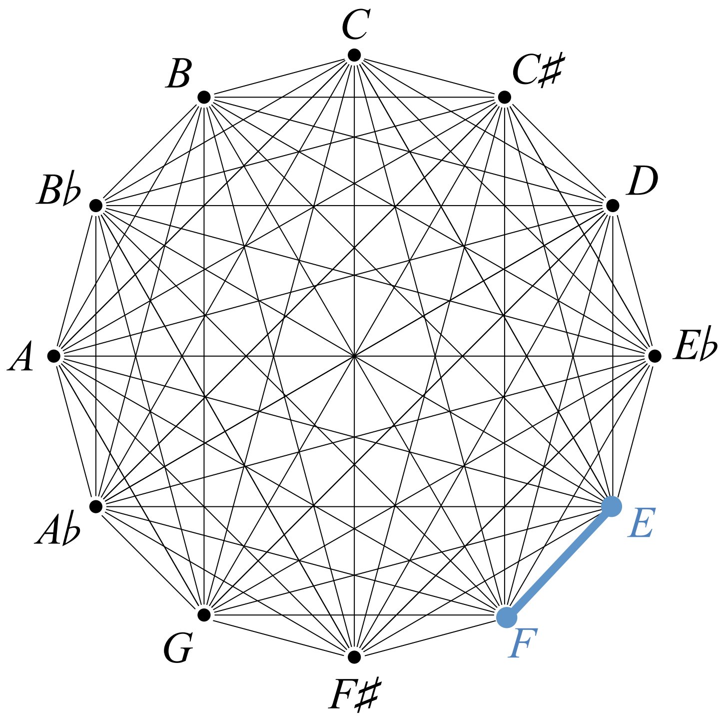
Complete graph K_{12}



`n0 < i1`

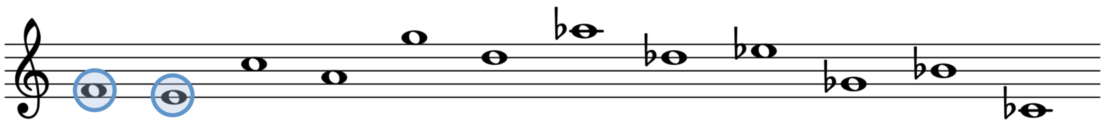
Spatial Interpretation of AIS

- AIS as *movement* in some *space*
- What kind of *space*?
  - Search space
    - Pitch classes: 0-cells
    - Intervals: 1-cells



Complete graph K_{12}

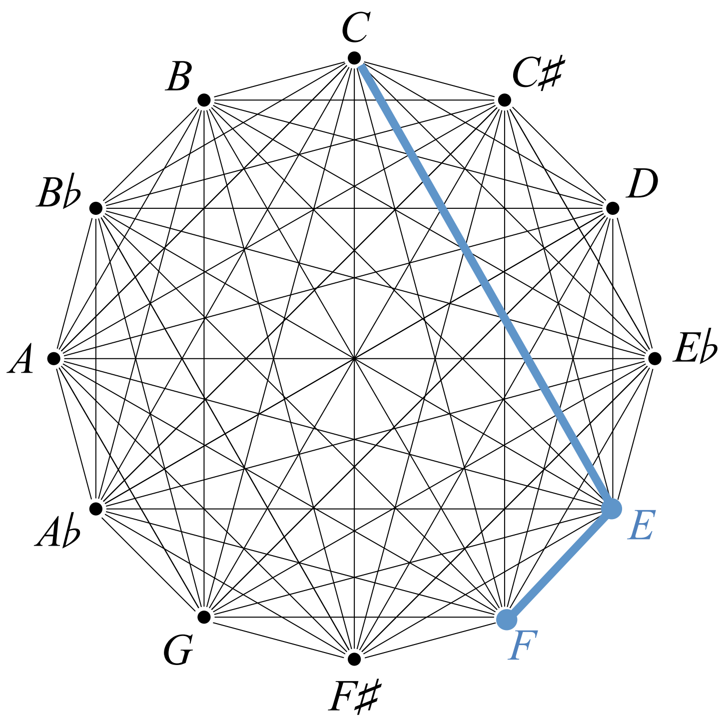
- What kind of *movement*?



`n0 < i1 > n1`

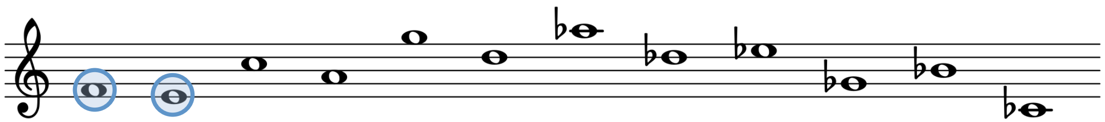
Spatial Interpretation of AIS

- AIS as *movement* in some *space*
- What kind of *space*?
    - Search space
        - Pitch classes: 0-cells
        - Intervals: 1-cells



Complete graph K_{12}

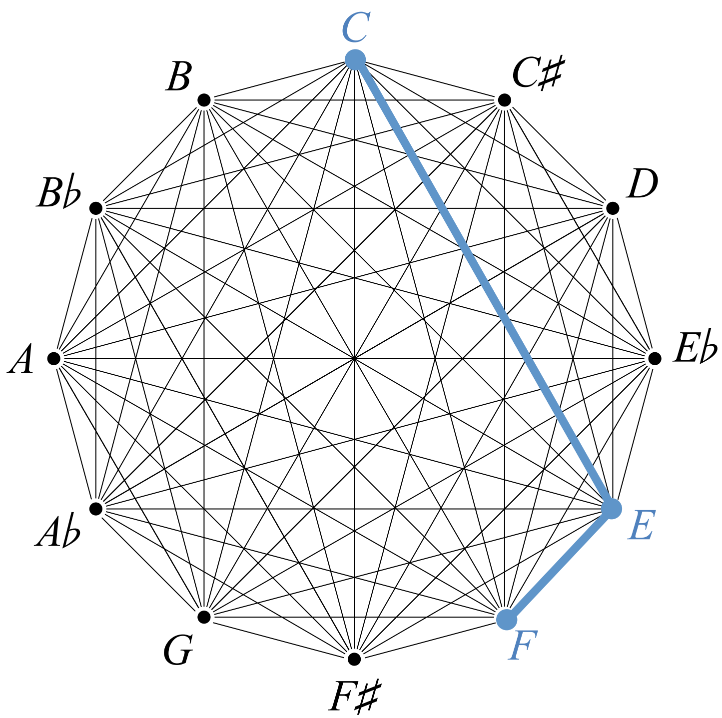
- What kind of *movement*?



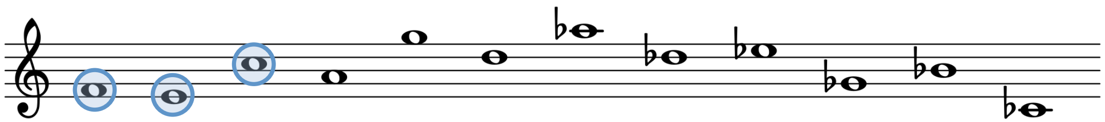
`n0 < i1 > n1 < i2`

Spatial Interpretation of AIS

- AIS as *movement* in some *space*
- What kind of *space*?
  - Search space
    - Pitch classes: 0-cells
    - Intervals: 1-cells
- What kind of *movement*?



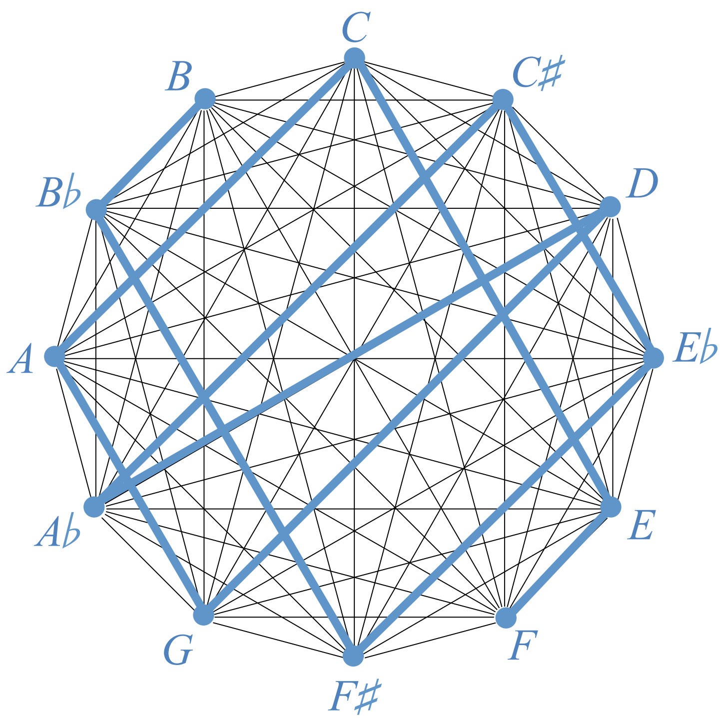
Complete graph K_{12}



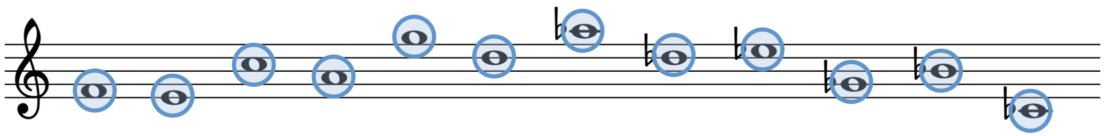
```
n0 < i1 > n1 < i2 > n2
```

Spatial Interpretation of AIS

- AIS as *movement* in some *space*
- What kind of *space*?
  - Search space
    - Pitch classes: 0-cells
    - Intervals: 1-cells
- What kind of *movement*?



Complete graph K_{12}

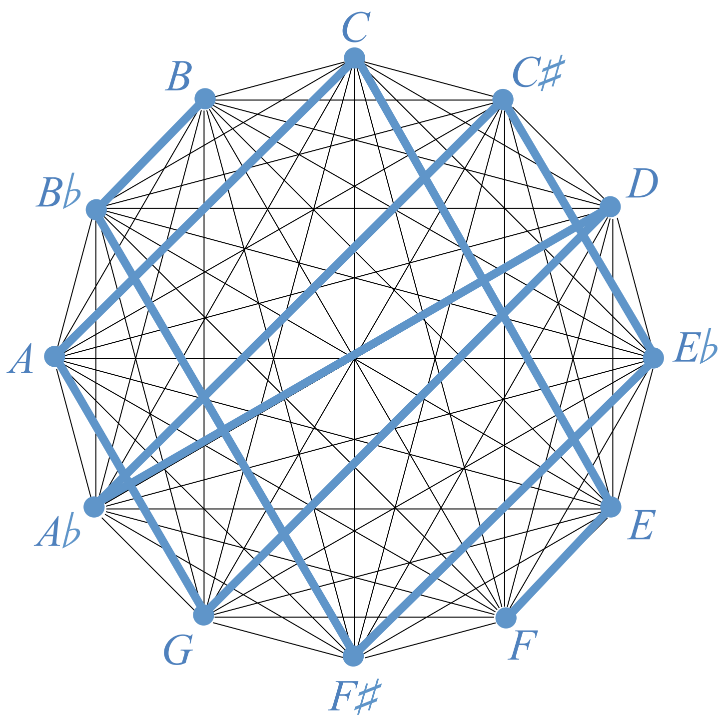


Hamiltonian path

`n0 < i1 > n1 < i2 > n2 < … > n10 < i11 > n11`

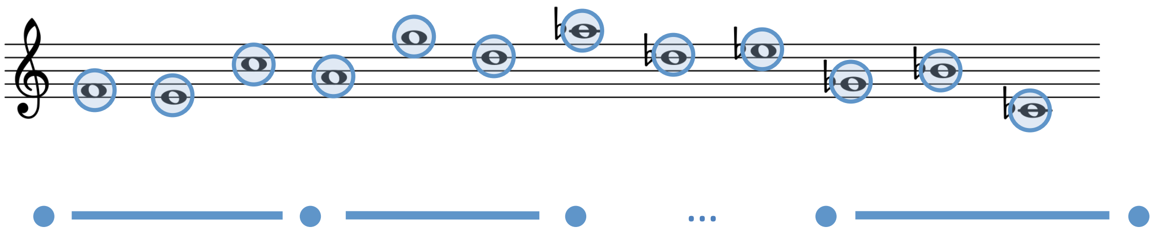
Spatial Interpretation of AIS

- AIS as *movement* in some *space*
- What kind of *space*?
  - Search space
    - Pitch classes: 0-cells
    - Intervals: 1-cells



Complete graph K_{12}

- What kind of *movement*?



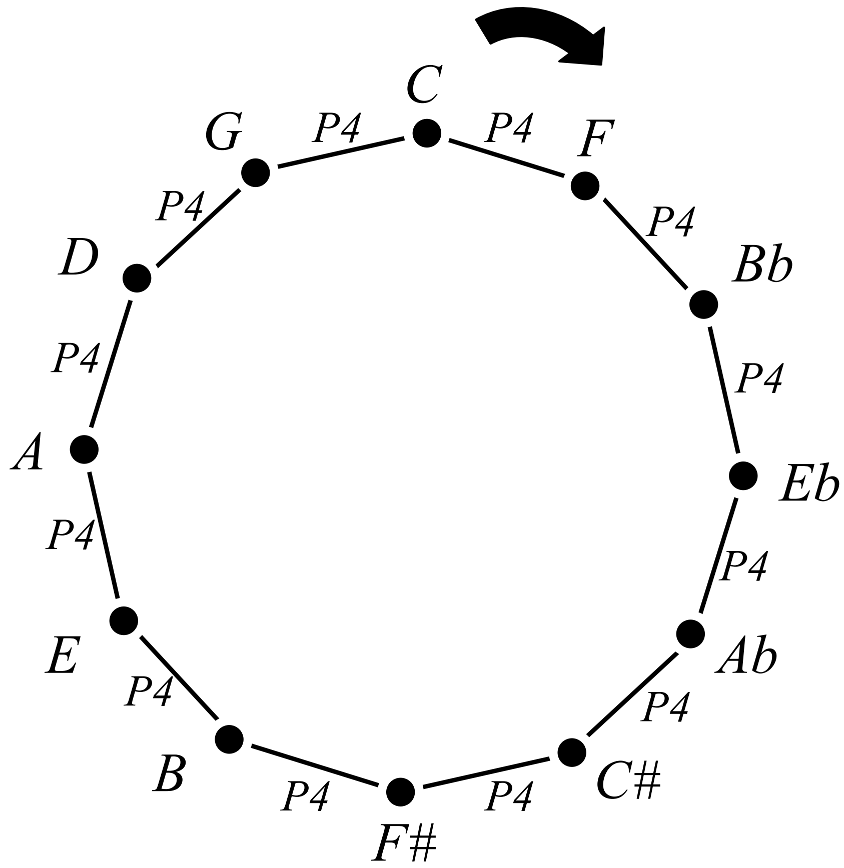
Hamiltonian path

such that edges have different labels

```
n0 < i1 > n1 < i2 > n2 < … > n10 < i11 > n11 / unique(i1,i2,…,i11)
```

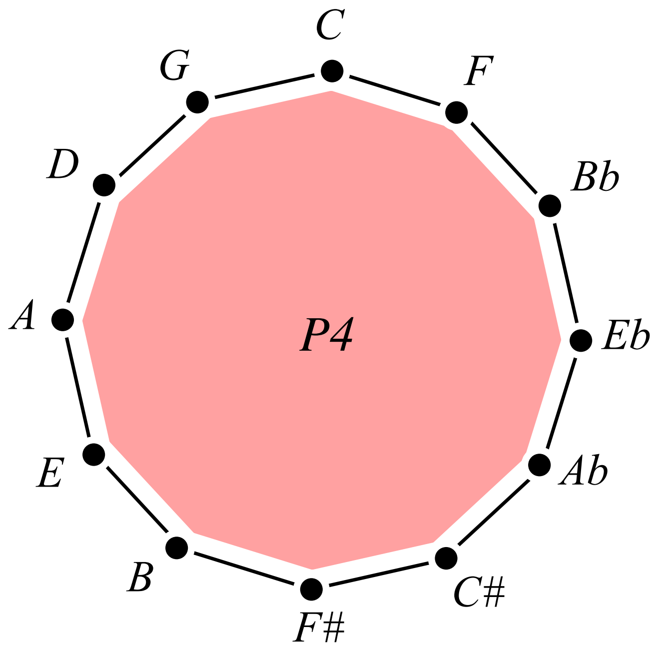
Spatial Interpretation of AIS

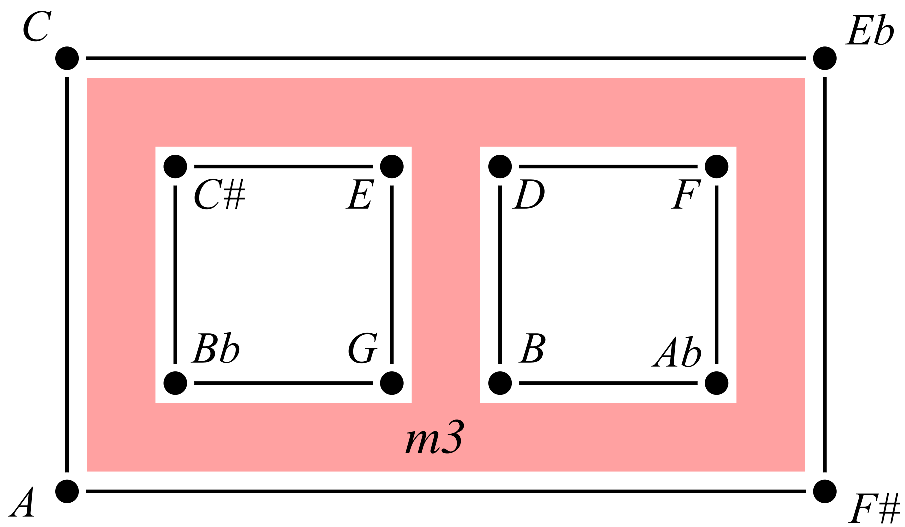
- AIS as *movement* in some *space*
- What kind of *space*?
  - Search space
    - Pitch classes: 0-cells
    - Intervals: 1-cells (not unique)
    - **Interval classes: 2-cells**
- What kind of *movement*?



Spatial Interpretation of AIS

- AIS as *movement* in some *space*
- What kind of *space*?
  - Search space
    - Pitch classes: 0-cells
    - Intervals: 1-cells (not unique)
    - **Interval classes: 2-cells**
- What kind of *movement*?





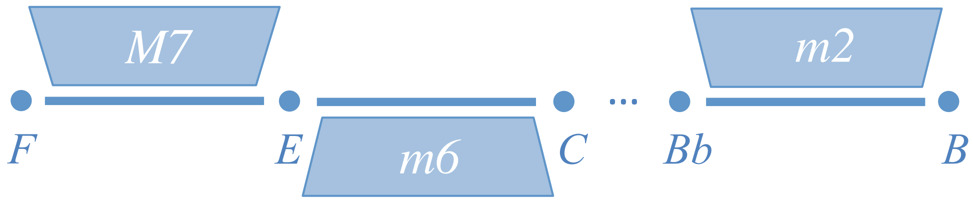
Spatial Interpretation of AIS

- AIS as *movement* in some *space*
- What kind of *space*?
  - Search space
    - Pitch classes: 0-cells
    - Intervals: 1-cells (not unique)
    - **Interval classes: 2-cells**
- What kind of *movement*?

AIS Complex
(too difficult to draw)

<0, 1>-*Hamiltonian path*

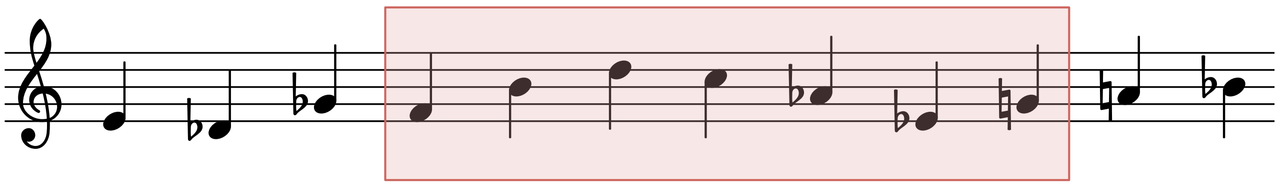
<0, 2>-*Eulerian path*



`n0 < i1 > n1 >` … `n10 < i11 < I11 > i11 > n11`

Intermediate Summary

- Proof of concept
  - Spatial reformulation of an old music problem
  - Geometrical characterization of AIS
  - Structural (spatial) expression of constraints
- Other contributions
  - Topological classification of AIS based on the AIS complex
  - New method to build AIS including particular patterns



Harmonic C minor scale

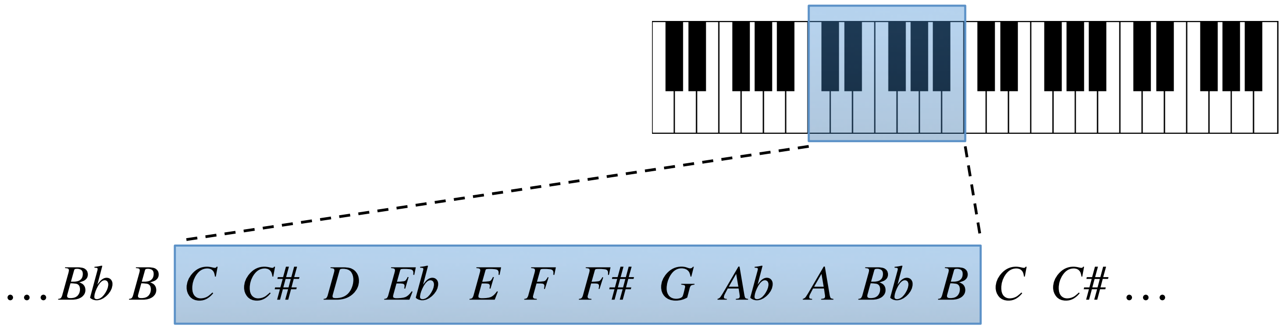
Outline

Bridging the gap between spatial computing and music theory

1. Proof of concept: a spatial study of all-interval series
2. Building chord spaces for music theory and analysis
3. Linking spaces for music generation and analysis
4. Conclusion and perspectives

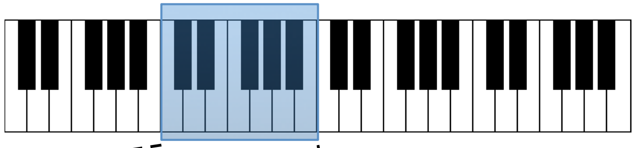
Musical Representations

- *Set Theory*
  - Equal temperament
  - Octave reduction according to a scale

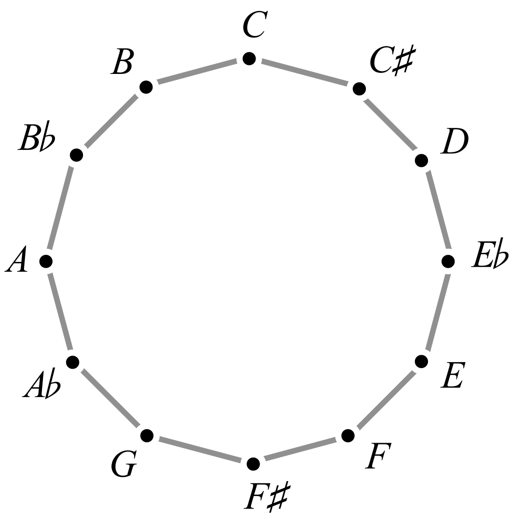


Musical Representations

- *Set Theory*
  - Equal temperament
  - Octave reduction according to a scale

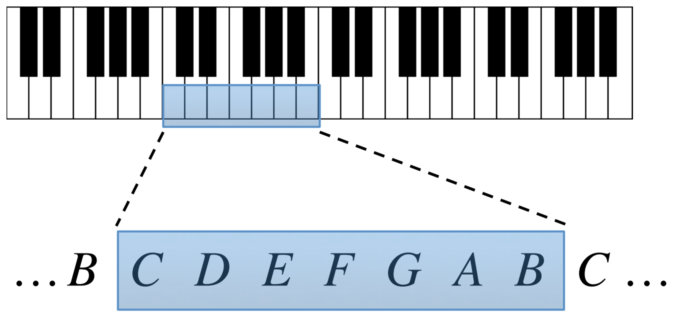


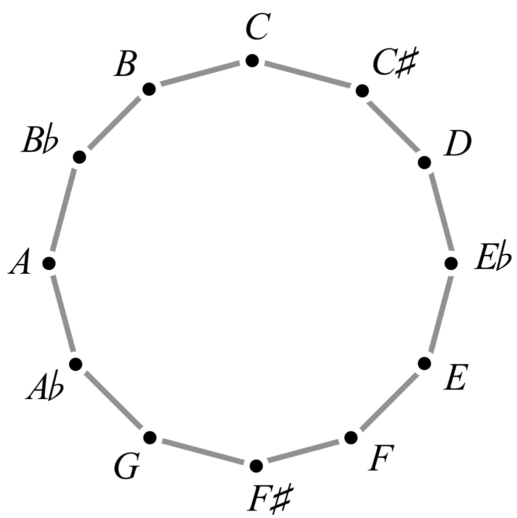
… Bb B C C# D Eb E F F# G Ab A Bb B C C# …

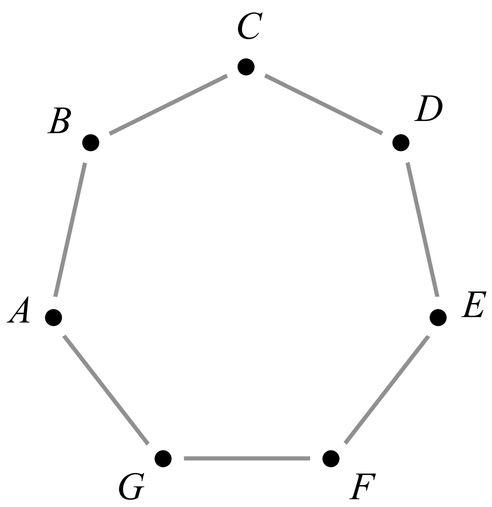


Musical Representations

- *Set Theory*
  - Equal temperament
  - Octave reduction according to a scale

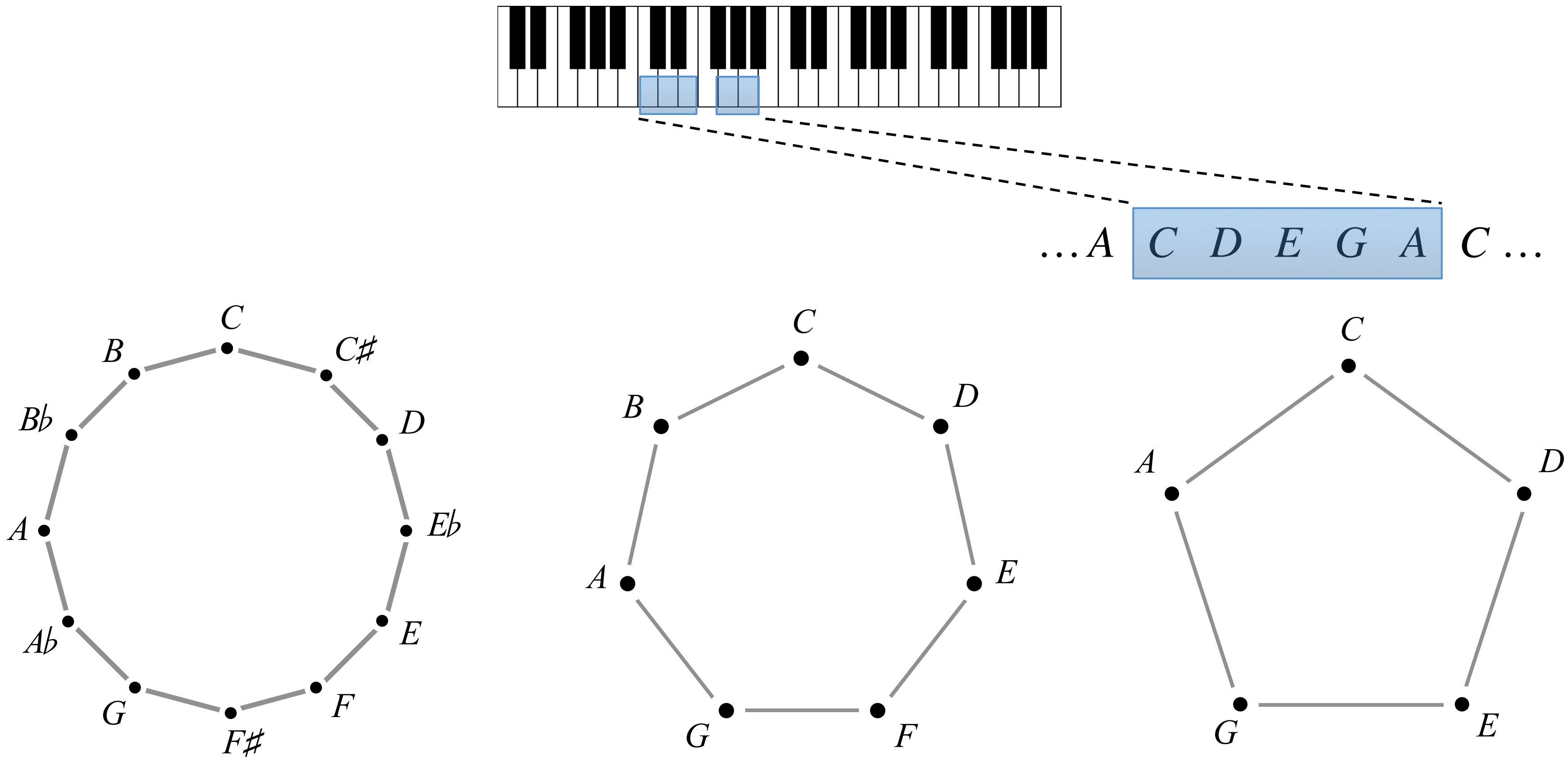






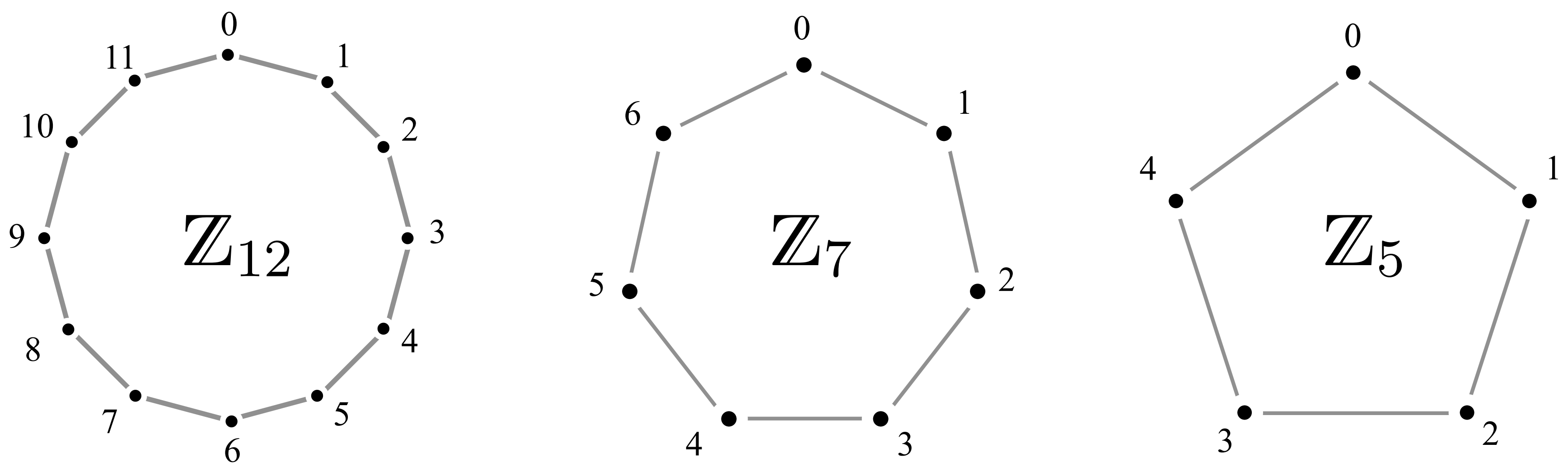
Musical Representations

- *Set Theory*
  - Equal temperament
  - Octave reduction according to a scale



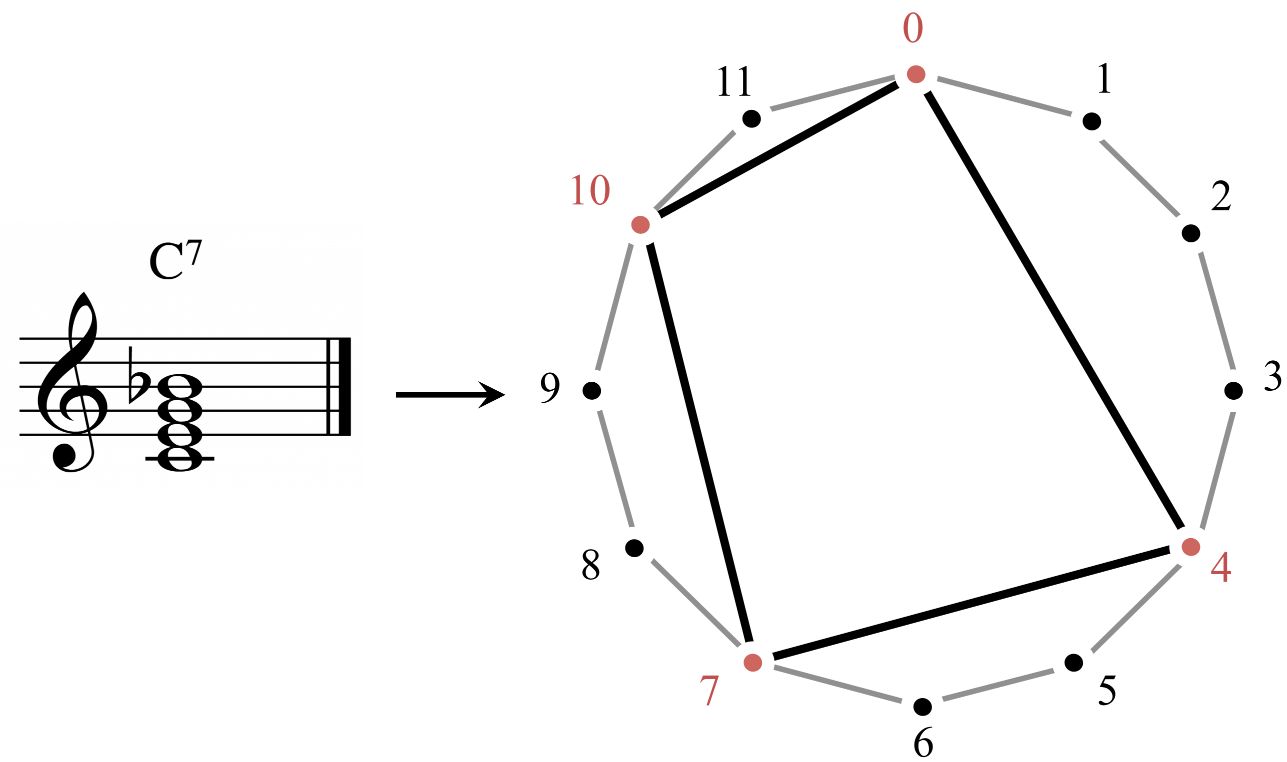
Musical Representations

- *Set Theory*
  - Equal temperament
  - Octave reduction according to a scale
  - Underlying structure of the cyclic group $\mathbb{Z}_N$



Musical Representations

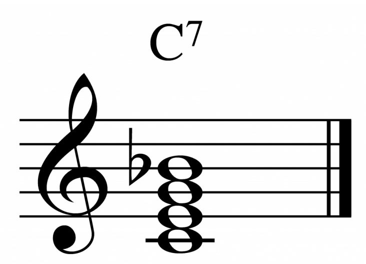
- *Set Theory*
  - Equal temperament
  - Octave reduction according to a scale
  - Underlying structure of the cyclic group $\mathbb{Z}_N$

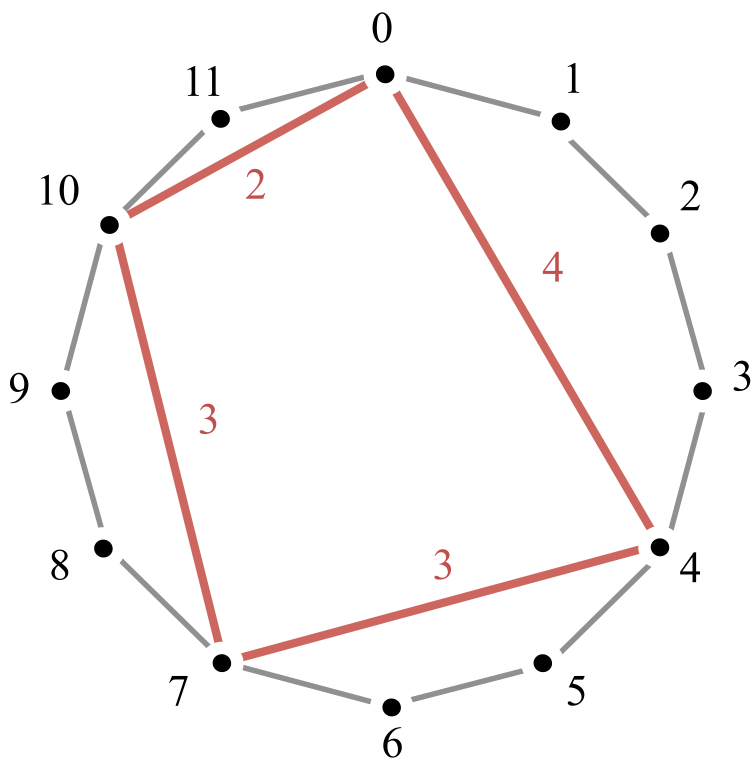


C = {0,4,7,10}

Musical Representations

- *Set Theory*
  - Equal temperament
  - Octave reduction according to a scale
  - Underlying structure of the cyclic group $\mathbb{Z}_N$

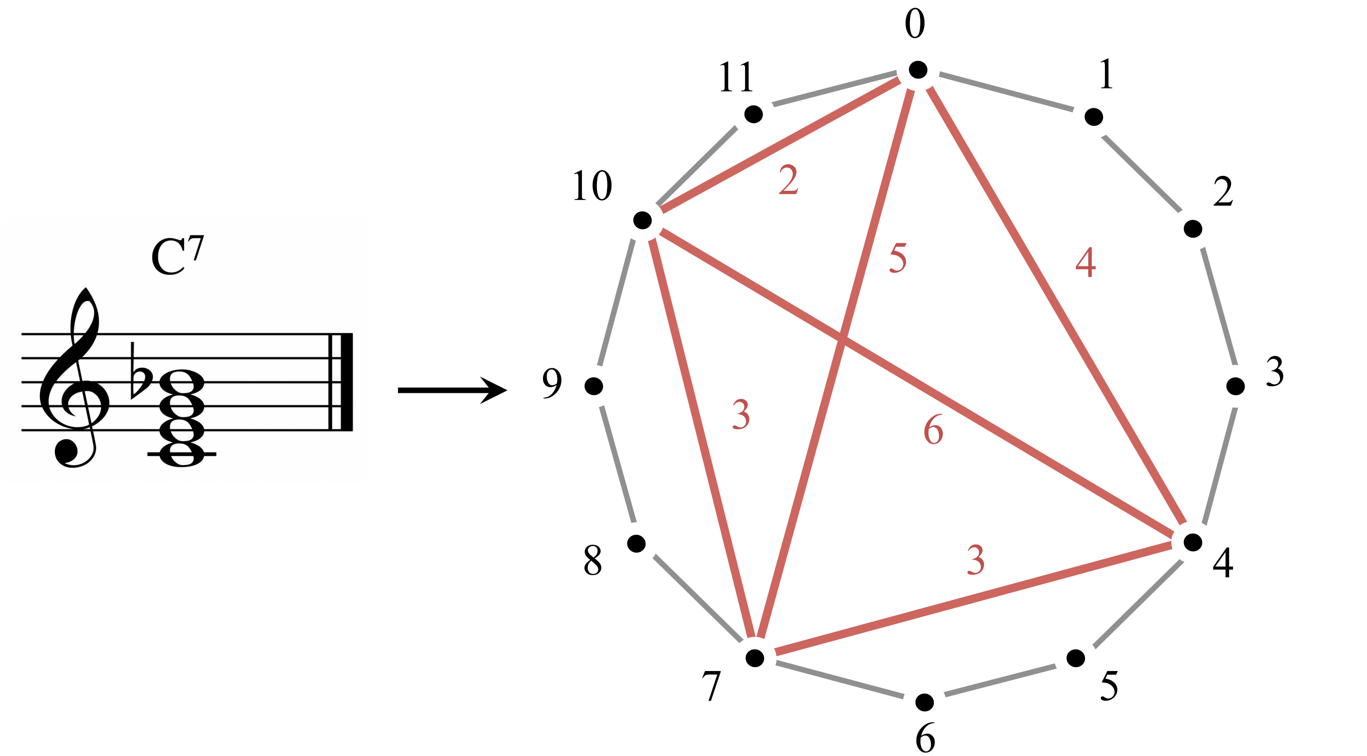




C = {0,4,7,10}
SI = [4,3,3,2]

Musical Representations

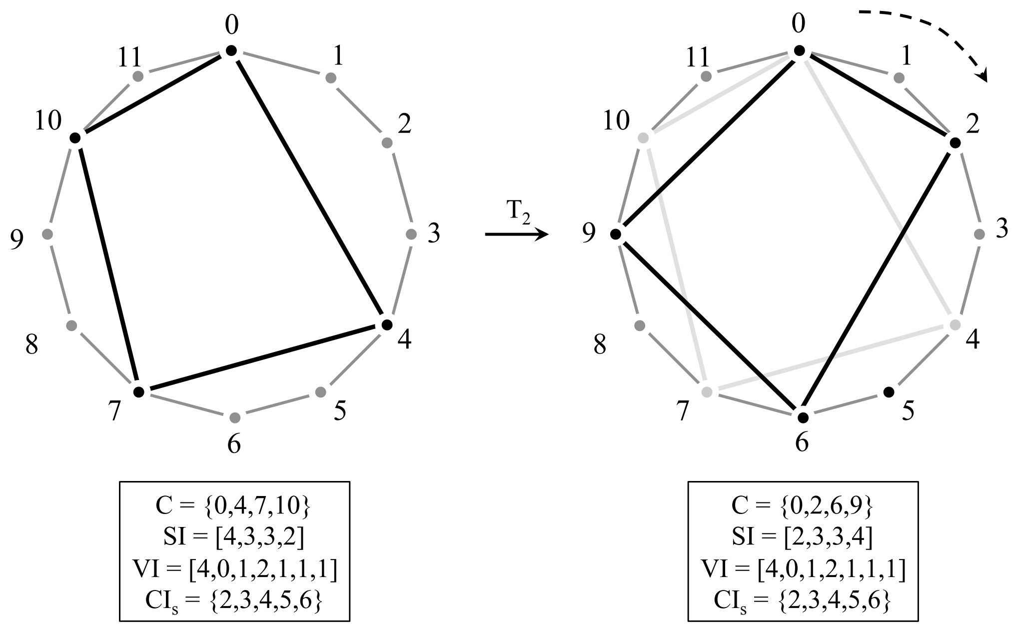
- *Set Theory*
  - Equal temperament
  - Octave reduction according to a scale
  - Underlying structure of the cyclic group $\mathbb{Z}_N$



$C = \{0,4,7,10\}$
$SI = [4,3,3,2]$
$CI_s = \{2,3,4,5,6\}$

Musical Representations

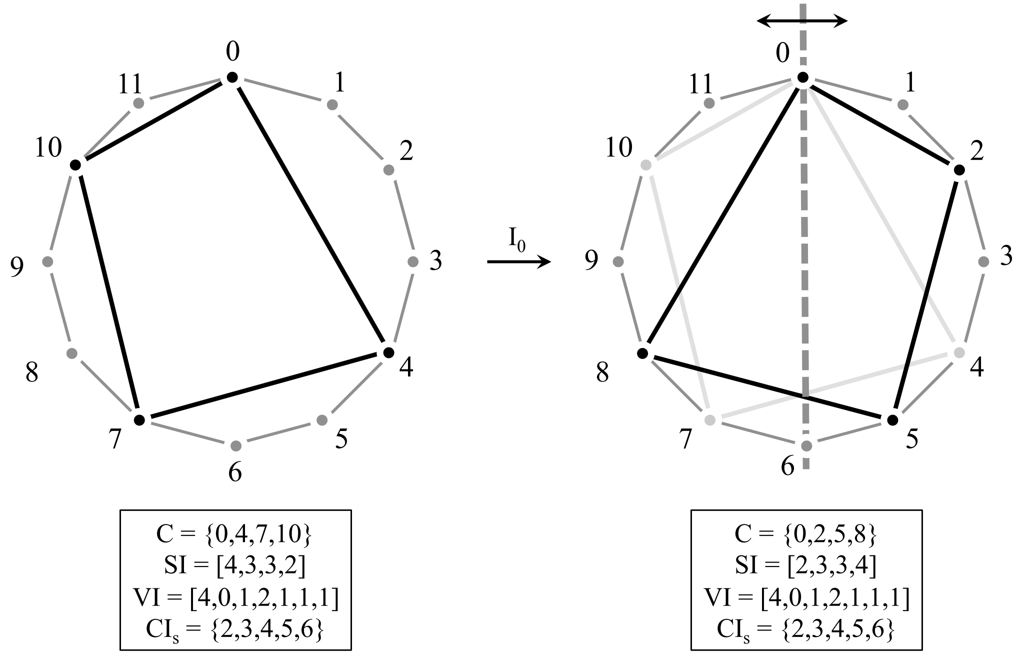
- *Set Theory*
  - Equal temperament
  - Octave reduction according to a scale
  - Underlying structure of the cyclic group $\mathbb{Z}_N$
  - Operations
    - Transpositions



$T_k : x \rightarrow x+k \bmod 12$

Musical Representations

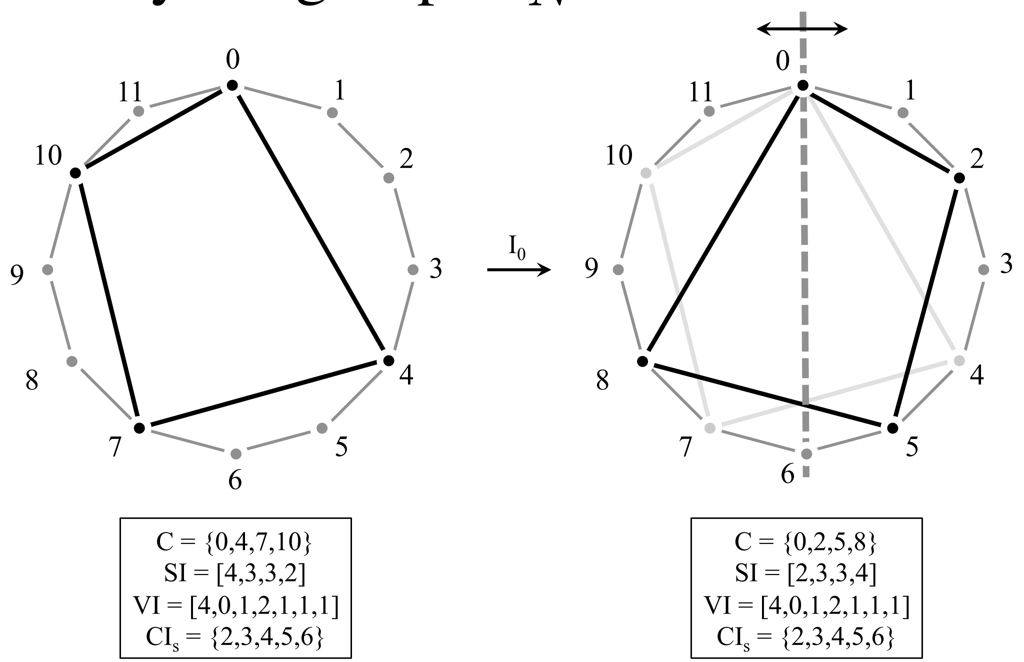
- *Set Theory*
  - Equal temperament
  - Octave reduction according to a scale
  - Underlying structure of the cyclic group $\mathbb{Z}_N$
  - Operations
    - Transpositions
    - Inversions



$$\mathrm{I} : x \rightarrow -x \bmod 12$$

Musical Representations

- *Set Theory*
  - Equal temperament
  - Octave reduction according to a scale
  - Underlying structure of the cyclic group $\mathbb{Z}_N$
  - Operations
    - Transpositions
    - Inversions
    - Permutations
    - Affine transformations



$$\mathrm{I} : x \rightarrow -x \bmod 12$$

Building Chord Complexes

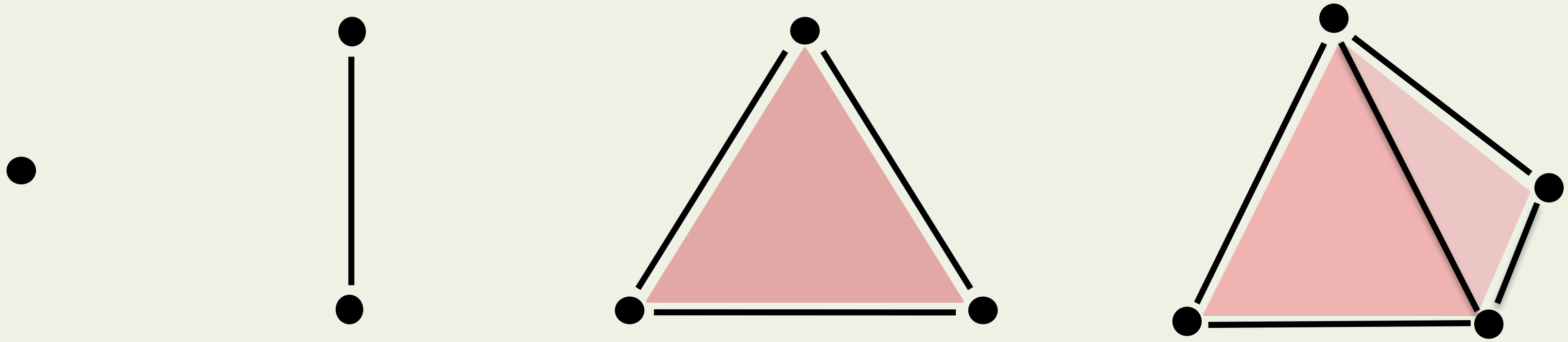
- Spatialization of chord sets

  Defining a topological collection to represent a set of chords

Building Chord Complexes

Toolbox: *Simplicial complex*

- *n*-Simplex: complete cellular complex from $n + 1$ vertices

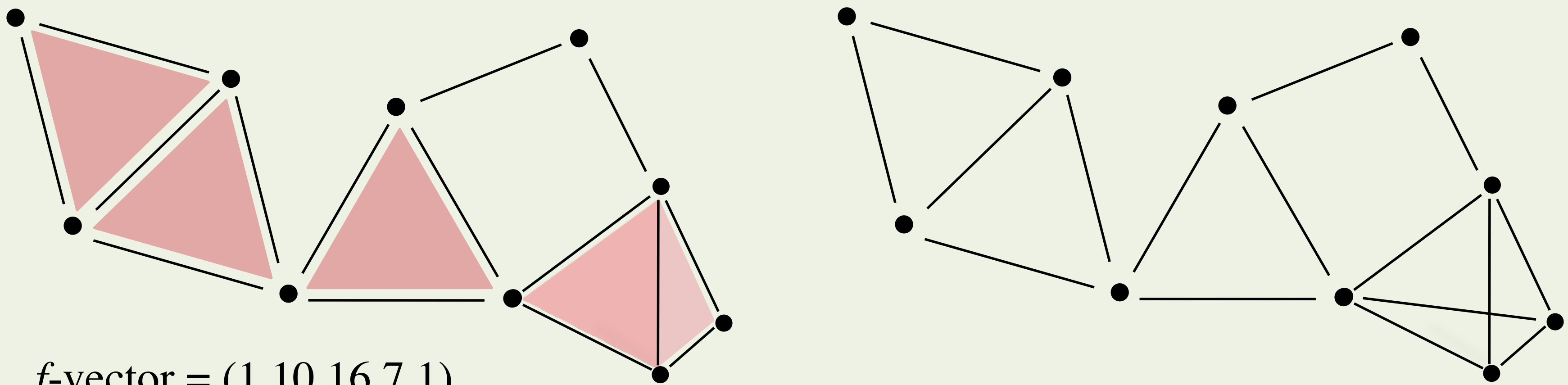


0-simplex 1-simplex 2-simplex 3-simplex

Building Chord Complexes

Toolbox: *Simplicial complex*

- n-Simplex: complete cellular complex from $n+1$ vertices
- n-Simplicial complex
  - Aggregate of p-simplicies ($p \leq n$)
  - f-vector ($f_0 = 1, f_1, \ldots, f_{n+1}$)
    f_{p+1} is the number of p-simplices
  - p-skeleton: simplices of dimension $\leq p$

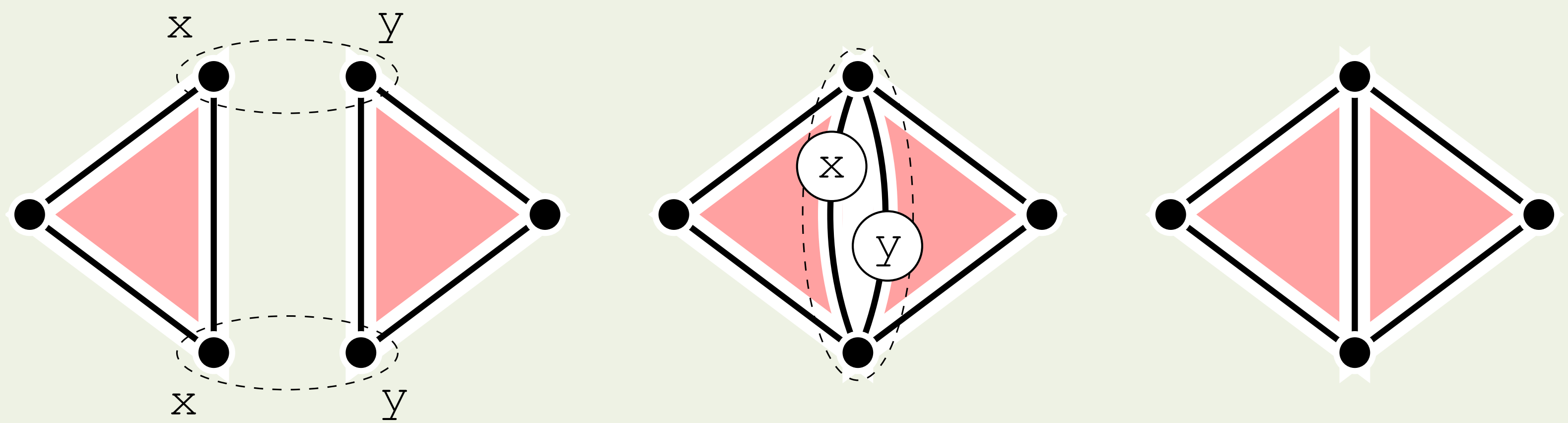


f-vector = (1,10,16,7,1)

Building Chord Complexes

Toolbox: *Self-Assembly*

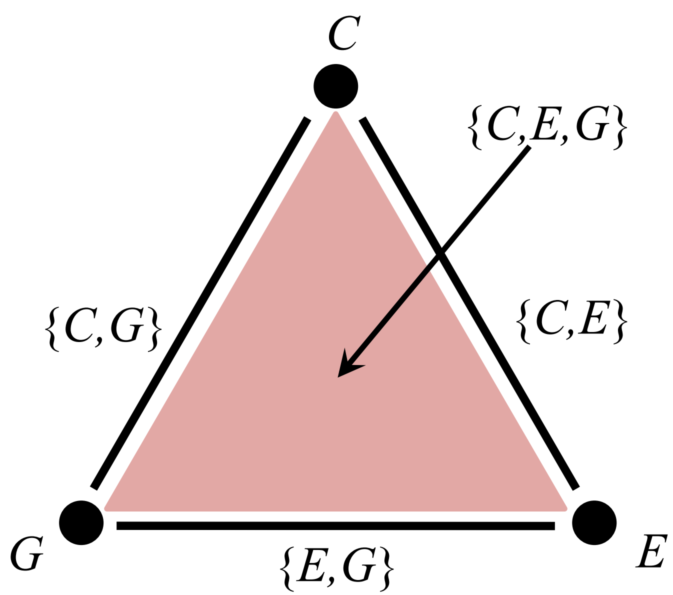
- Metaphor taken from chemical reactions
- Building simplicial collection from elementary parts

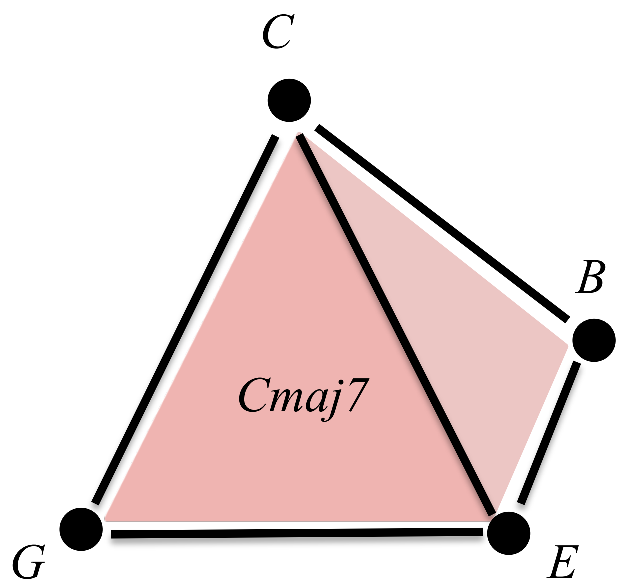



```
Transformation Self-Assembly =
    x y / (x == y) and (faces x = faces y) => let c = new_cell (dim x) (faces x)
                                                                (union (cofaces x)
                                                                       (cofaces y))
                                              in x * c
```

Building Chord Complexes

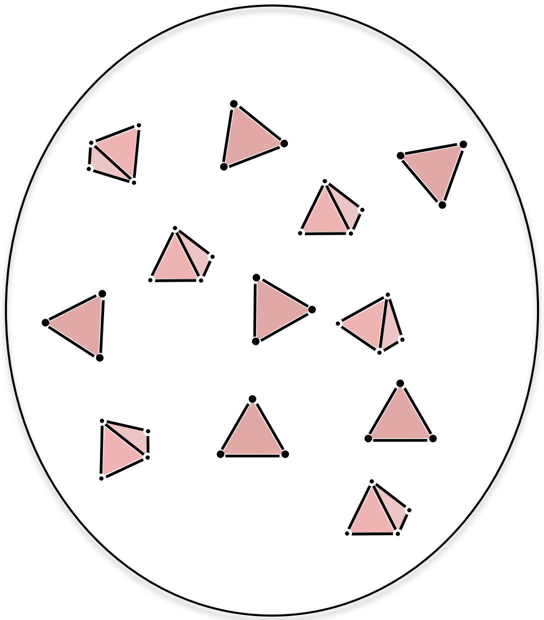
- Spatialization of chord sets
  - Simplicial representation of chords
    - 1-note chord: 0-simplex (vertex)
    - 2-note chord: 1-simplex (edge)
    - 3-note chord: 2-simplex (triangle)
    - 4-note chord: 3-simplex (tetrahedron)



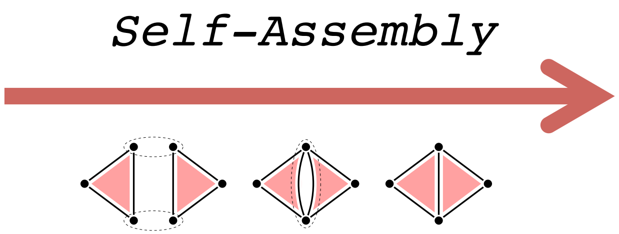


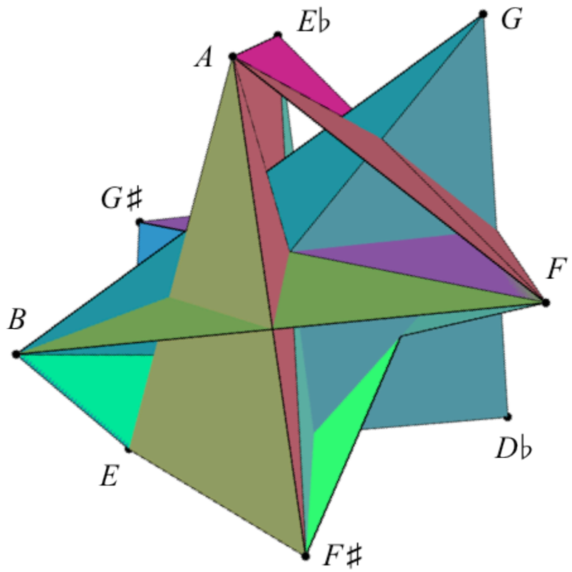
Building Chord Complexes

- Spatialization of chord sets
  - Simplicial representation of chords
  - Self-assembly of a set of chords



Initial population





Chord complex

Building Chord Complexes

- Spatialization of chord sets
  - Algebraic-based chord complexes

    Chord population from chord transformation orbits

    Transposition, inversion, permutation, etc.
  - Piece-based chord complexes

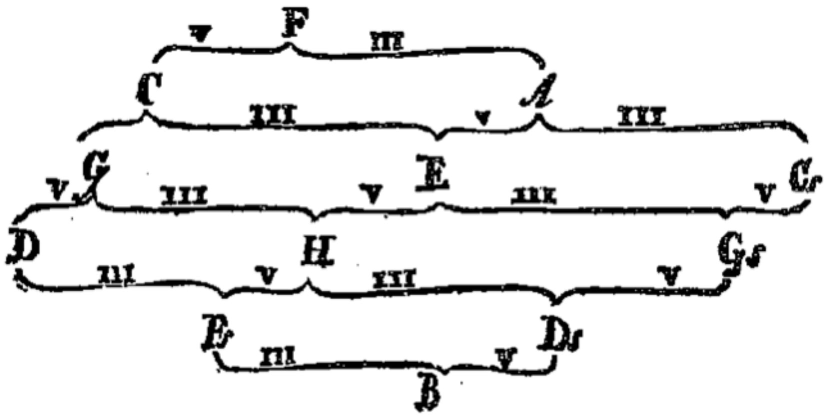
    Chord population from a piece segmentation

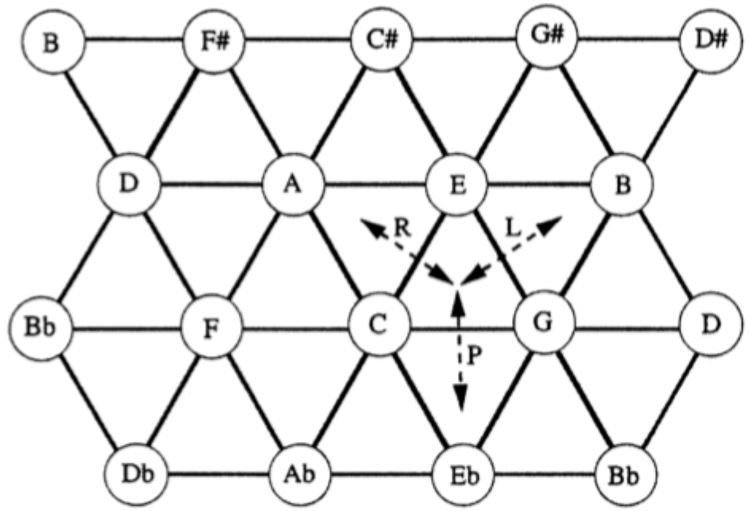
    Analysis of Bach, Chopin, Schoenberg, Webern, Glass, etc.

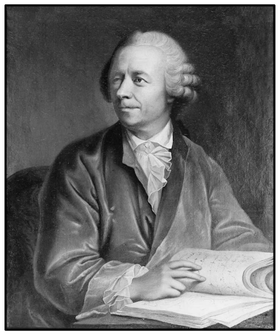
Algebraic-based Chord Complexes

- Motivation

  Formalization/generalization of the *Tonnetz*





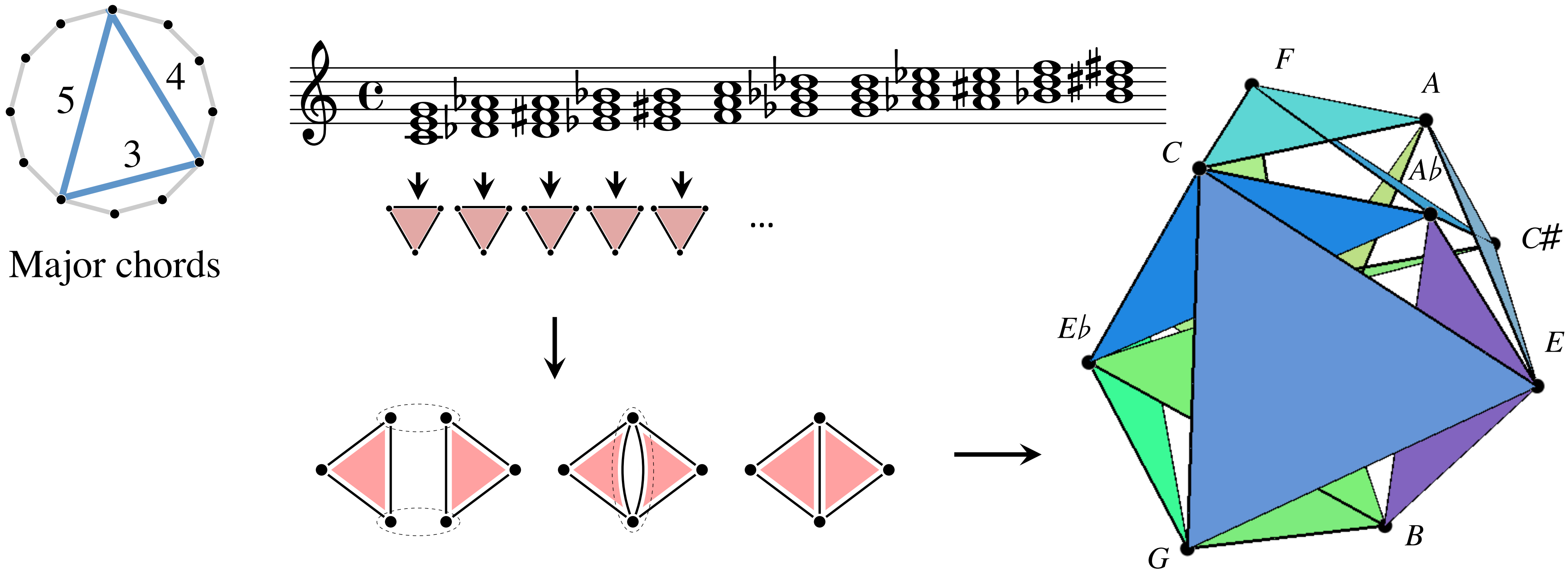


[Euler - 1739]

[Cohn - 1997]

Algebraic-based Chord Complexes

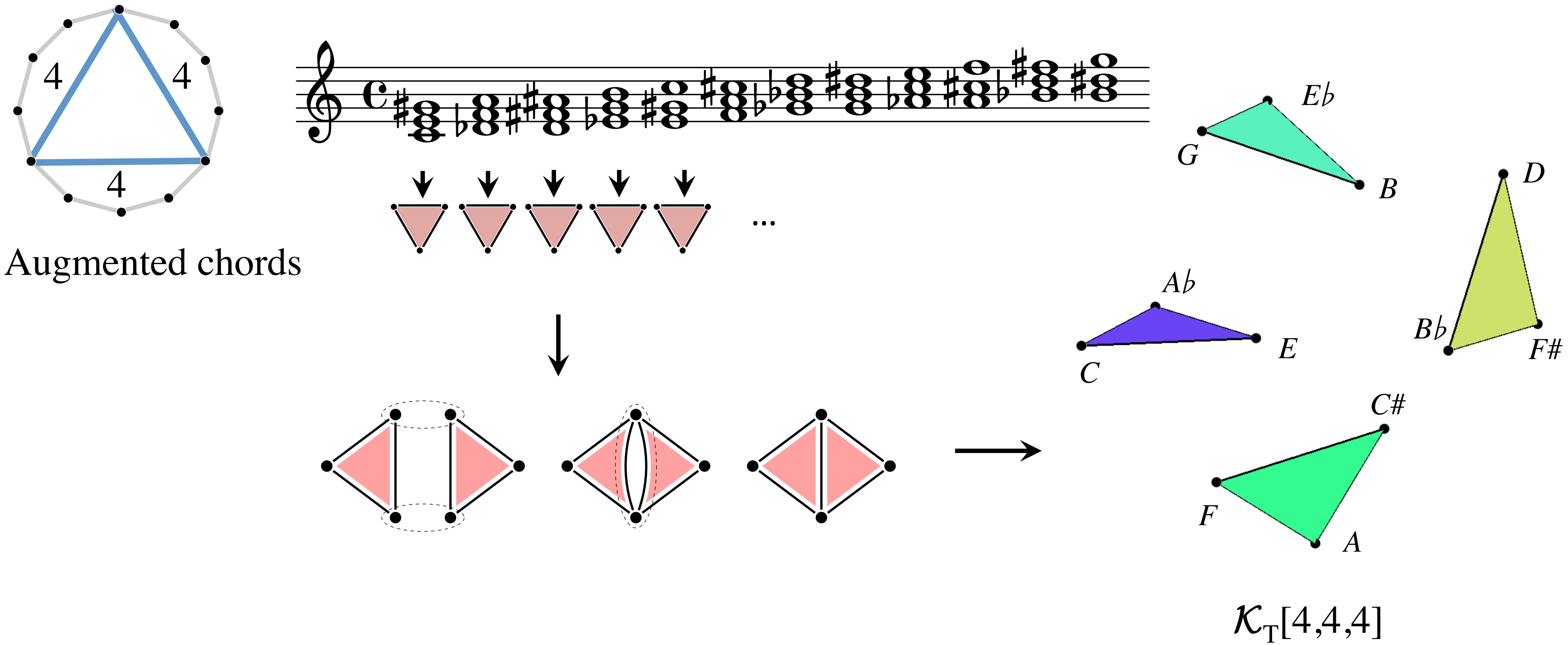
- Assembling chords related by some equivalence relation
  - Transposition (Cyclic group action $Z_n \rightarrow Z_n$)



$\mathcal{K}_T[4,3,5]$

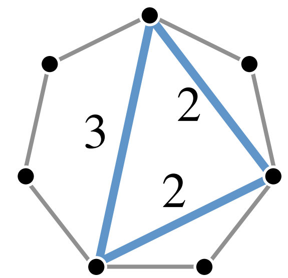
Algebraic-based Chord Complexes

- Assembling chords related by some equivalence relation
  - Transposition (Cyclic group action $Z_n \rightarrow Z_n$)

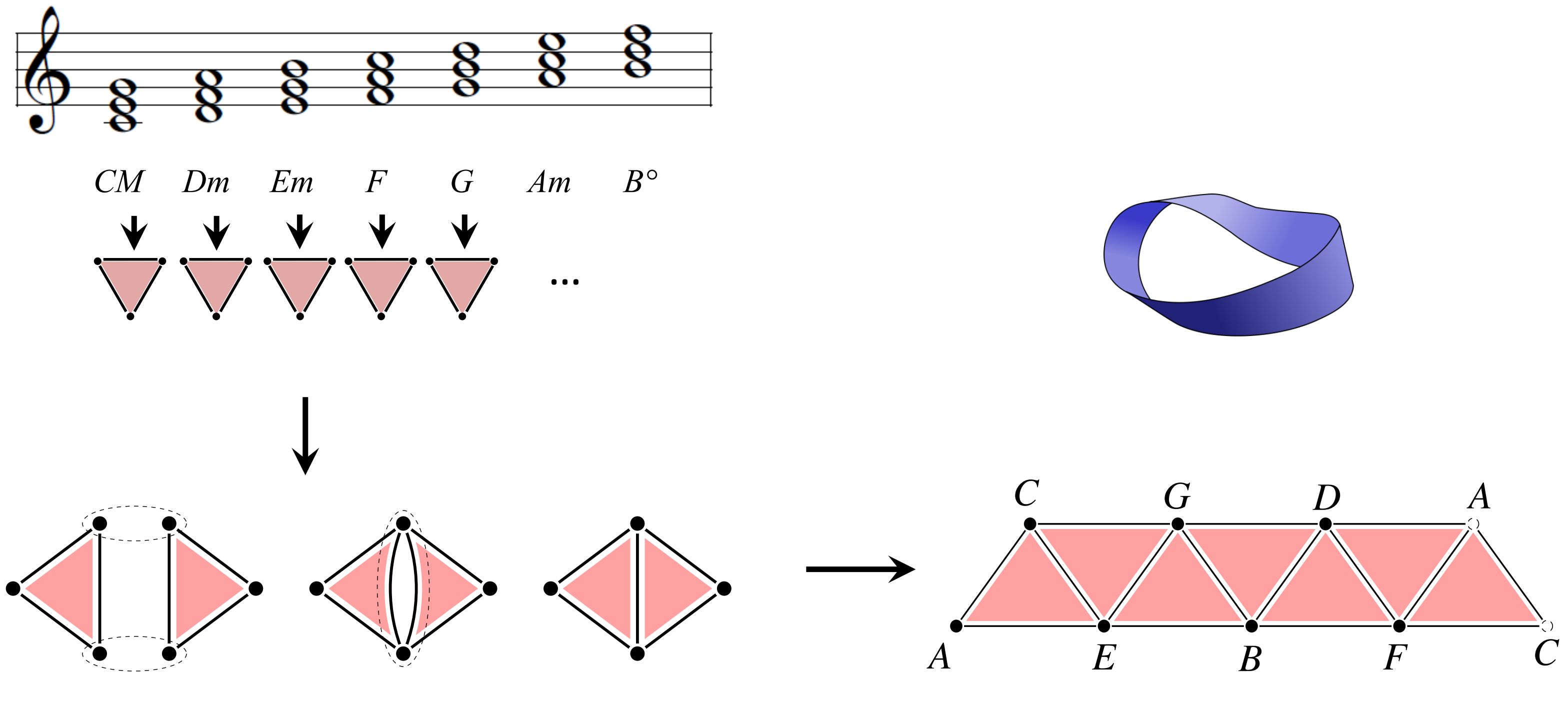


Algebraic-based Chord Complexes

- Assembling chords related by some equivalence relation
  - Transposition (Cyclic group action $Z_n \rightarrow Z_n$)



Diatonic triads

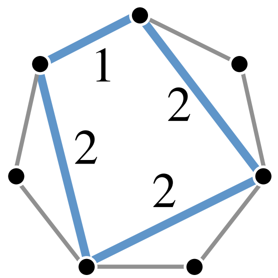


$\mathcal{K}_T[2,2,3]$

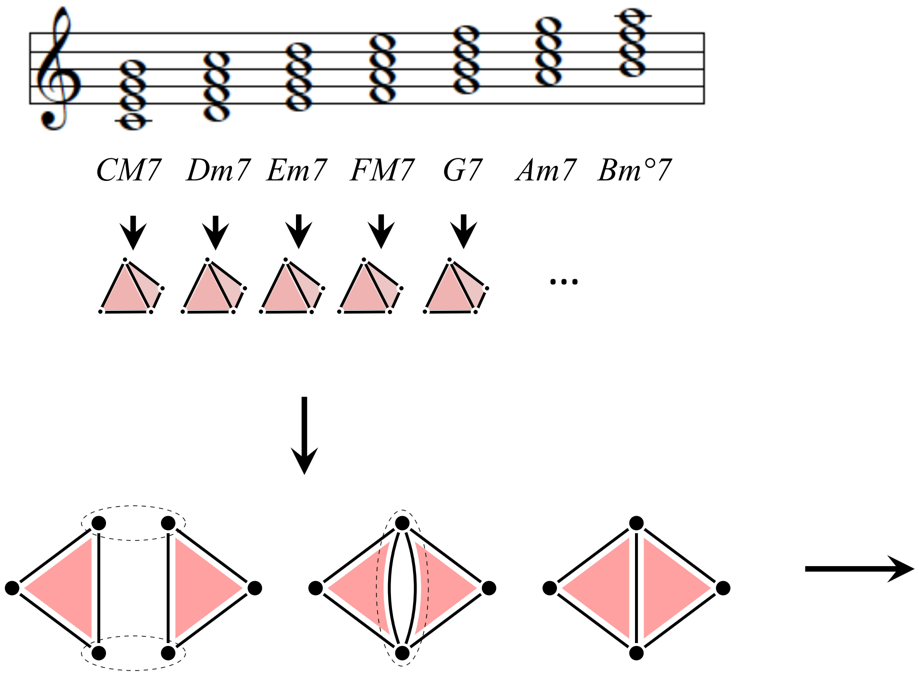
Mazzola, G. (2002). *The topos of music: geometric logic of concepts, theory, and performance*

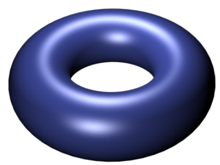
Algebraic-based Chord Complexes

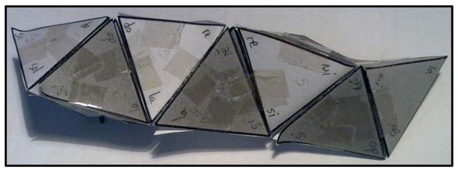
- Assembling chords related by some equivalence relation
  - Transposition (Cyclic group action $Z_n \rightarrow Z_n$)



Diatonic 4-degrees





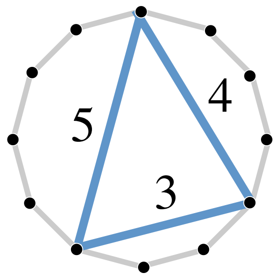


$\mathcal{K}_T[2,2,2,1]$

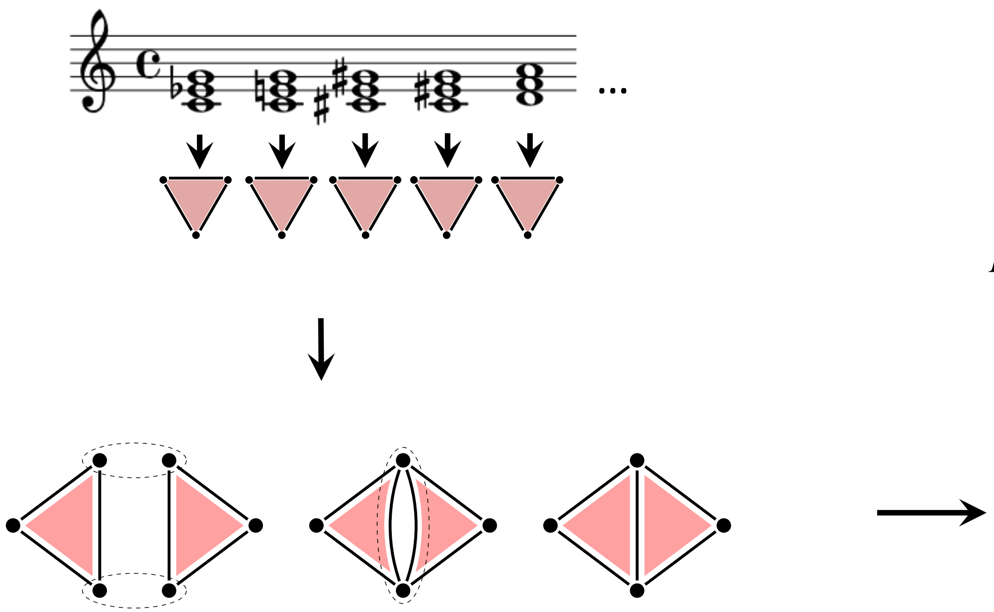
Bigo et al. (2011) – Building topological spaces for musical objects

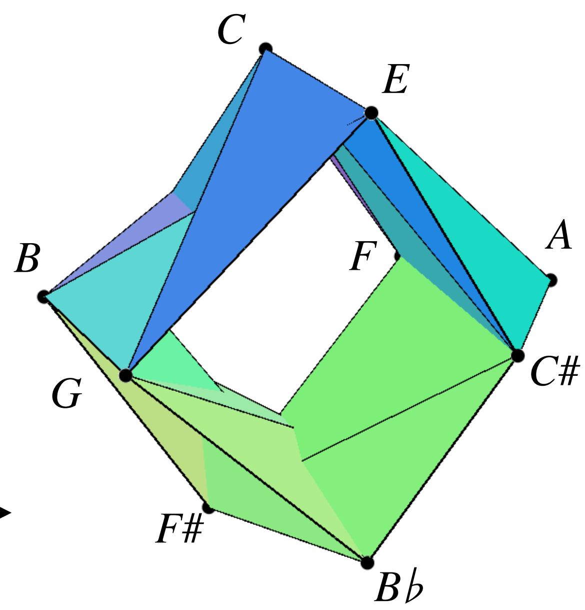
Algebraic-based Chord Complexes

- Assembling chords related by some equivalence relation
  - Transposition (Cyclic group action $Z_n \rightarrow Z_n$)
  - Transposition and inversion (Dihedral group action $D_n \rightarrow Z_n$)



major/minor triads





$\mathcal{K}_{\mathrm{TI}}[3,4,5]$

Algebraic-based Chord Complexes

- Complexes enumeration in heptatonic/chromatic systems

Equivalence relation	Chromatic system (Z_{12})	Heptatonic system (Z_7)
Transposition	352 complexes	20 complexes
Transposition / inversion	224 complexes	18 complexes
Permutations	77 complexes	16 complexes
Application affine	157 complexes	16 complexes

Algebraic-based Chord Complexes

- Complexes enumeration in heptatonic/chromatic systems

$\mathcal{S}_1(\mathcal{K}_{TI}[3,4,5])$
[Cohn – 1997]

$\mathcal{S}_1(\mathcal{K}_{TI}[2,3,3,4])$
[Gollin - 1998]

$\mathcal{K}_{T}[3,4]$, $\mathcal{K}_{T}[2,2,3]$, $\mathcal{K}_{T}[1,2,2,2]$
[Mazzola – 2002]

$\mathcal{K}_{TI}[1,1,10] \longrightarrow \mathcal{K}_{TI}[4,4,4]$
[Catanzaro - 2011]

$\mathcal{S}_1(\mathcal{K}_{TI}[1,2,4])$
[Hook – 2013]

d	complexe	taille	b_n	p-v	χ
-	$\mathcal{K}_\emptyset$	0	0		0
0	$\mathcal{K}_{TI}[0]$	0	[0]		0
1	$\mathcal{K}_{TI}[1,11]$	12	[1, 1]	x	0
	$\mathcal{K}_{TI}[2,10]$	12	[2, 2]		0
	$\mathcal{K}_{TI}[3,9]$	12	[3, 3]		0
	$\mathcal{K}_{TI}[4,8]$	12	[4, 4]		0
	$\mathcal{K}_{TI}[5,7]$	12	[1, 1]	x	0
	$\mathcal{K}_{TI}[6,6]$	6	[6, 0]		6
2	$\mathcal{K}_{TI}[1,1,10]$	12	[1, 1, 0]	x	0
	$\mathcal{K}_{TI}[1,2,9]$	24	[1, 2, 1]	x	0
	$\mathcal{K}_{TI}[1,3,8]$	24	[1, 2, 1]	x	0
	$\mathcal{K}_{TI}[1,4,7]$	24	[1, 2, 1]	x	0
	$\mathcal{K}_{TI}[1,5,6]$	24	[1, 1, 6]		6
	$\mathcal{K}_{TI}[2,2,8]$	12	[2, 2, 0]		0
	$\mathcal{K}_{TI}[2,3,7]$	24	[1, 2, 1]	x	0
	$\mathcal{K}_{TI}[2,4,6]$	24	[2, 2, 6]		6
	$\mathcal{K}_{TI}[2,5,5]$	12	[1, 1, 0]	x	0
	$\mathcal{K}_{TI}[3,3,6]$	12	[3, 0, 3]		6
	$\mathcal{K}_{TI}[3,4,5]$	24	[1, 2, 1]	x	0
	$\mathcal{K}_{TI}[4,4,4]$	4	[4, 0, 0]		4
	$\mathcal{K}_{TI}[1,1,1,9]$	12	[1, 1, 0, 0]	x	0
	$\mathcal{K}_{TI}[1,1,2,8]$	24	[1, 1, 12, 0]		12
	$\mathcal{K}_{TI}[1,1,3,7]$	24	[1, 2, 13, 0]		12
	$\mathcal{K}_{TI}[1,1,4,6]$	24	[1, 1, 18, 0]		18
	$\mathcal{K}_{TI}[1,1,5,5]$	12	[1, 1, 6, 0]		6

Building Chord Complexes

- Spatialization of chord sets

  - Algebraic-based chord complexes

    Chord population from chord transformation orbits

    Transposition, inversion, permutation, etc.

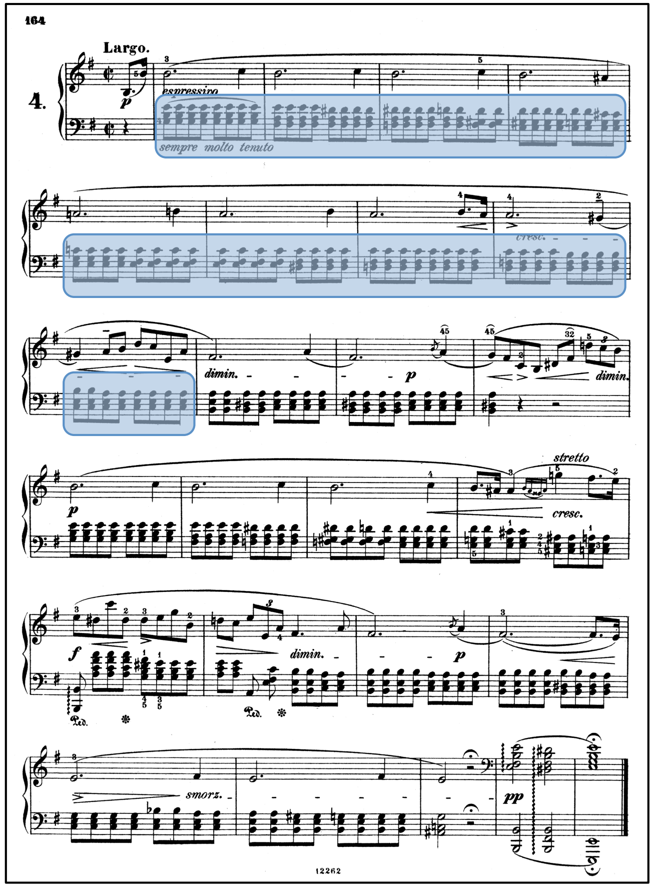
  - Piece-based chord complexes

    Chord population from a piece segmentation

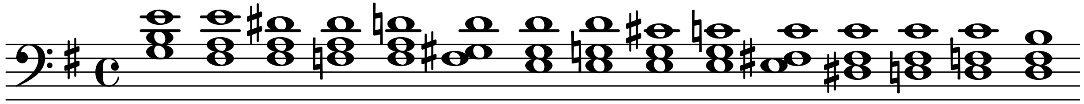
    Analysis of Bach, Chopin, Schoenberg, Webern, Glass, etc.

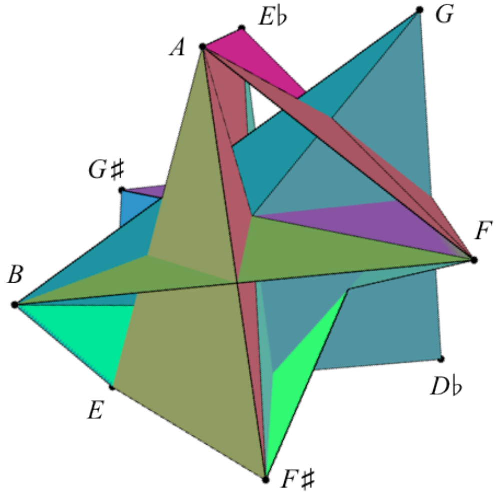
Piece-based Chord Complexes

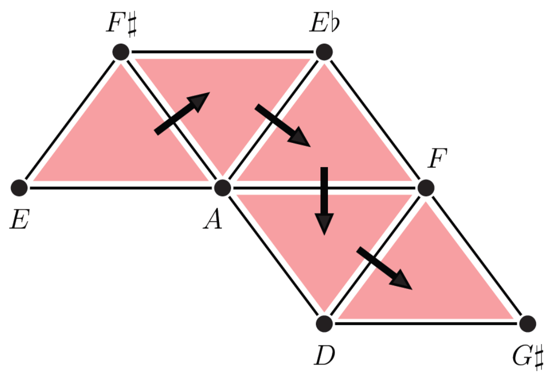
- Chord complex built from a piece



Prelude No. 4 Op. 28 of F. Chopin



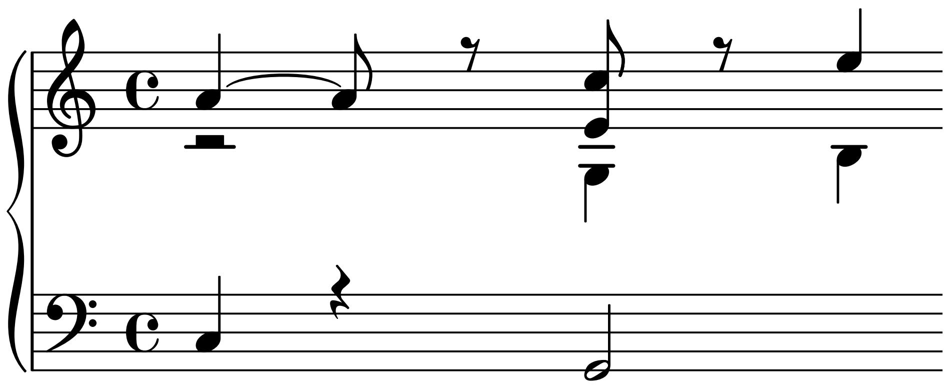




120 (2,1)-Hamiltonian paths

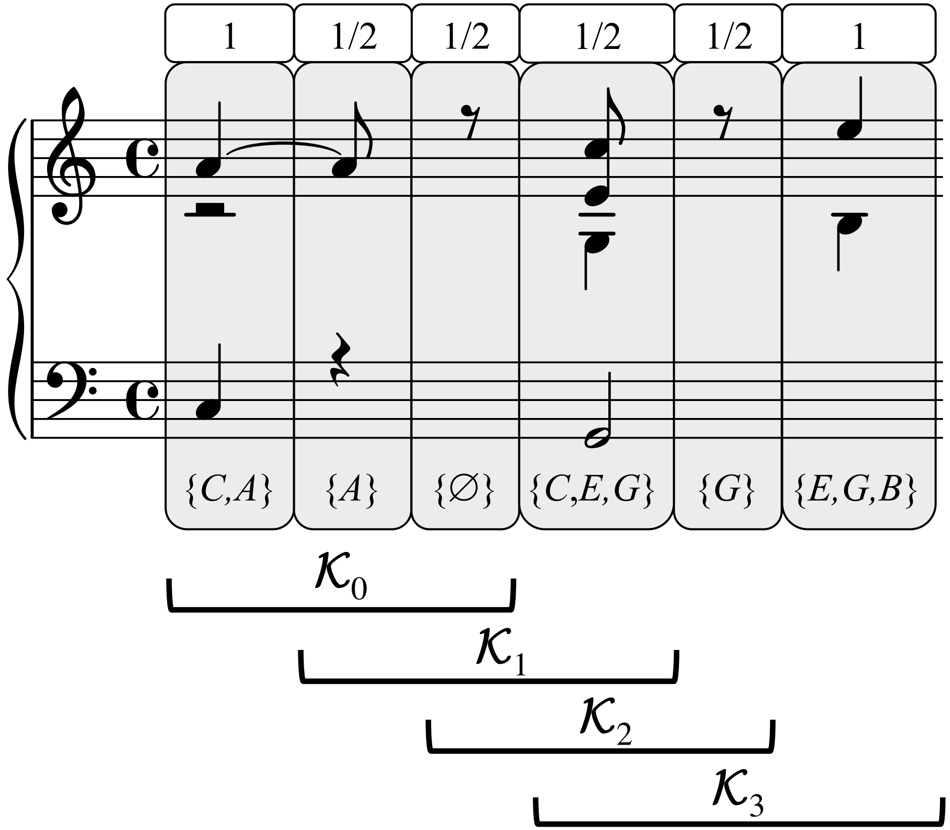
Piece-based Chord Complexes

- Chord complex built from a piece
- Sequence of complexes resulting from a segmentation



Piece-based Chord Complexes

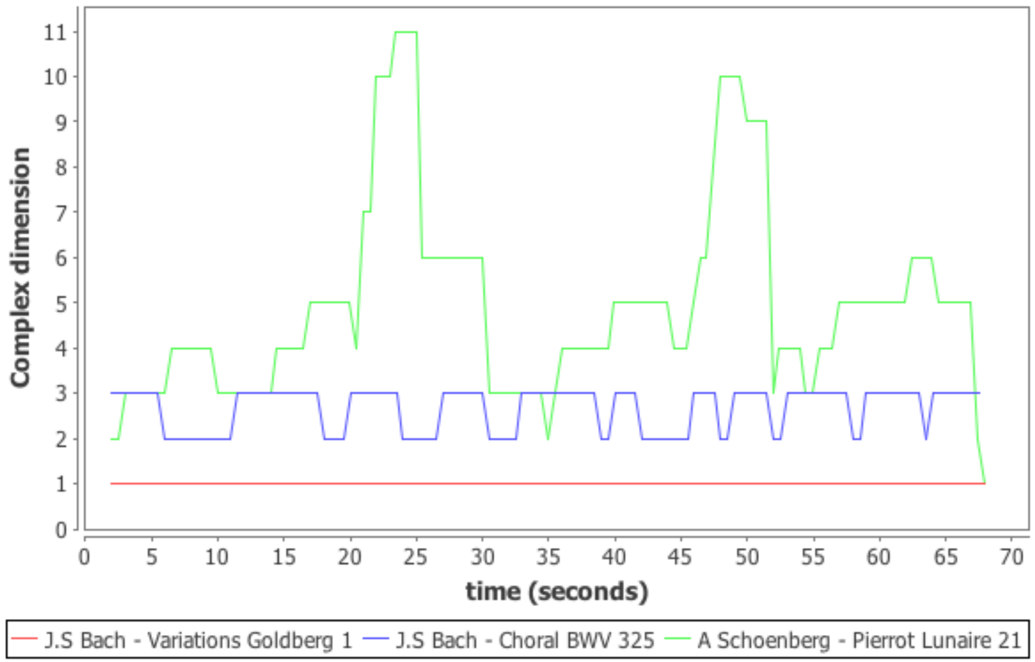
- Chord complex built from a piece
- Sequence of complexes resulting from a segmentation



Piece-based Chord Complexes

- Chord complex built from a piece
- Sequence of complexes resulting from a segmentation
- Meaningful musical properties via topological properties
  - Dimension

Number of distinct pitch classes simultaneously used in the same segment



Piece-based Chord Complexes

- Chord complex built from a piece
- Sequence of complexes resulting from a segmentation
- Meaningful musical properties via topological properties?
  - Dimension
  - Size (at some dimension *n*)

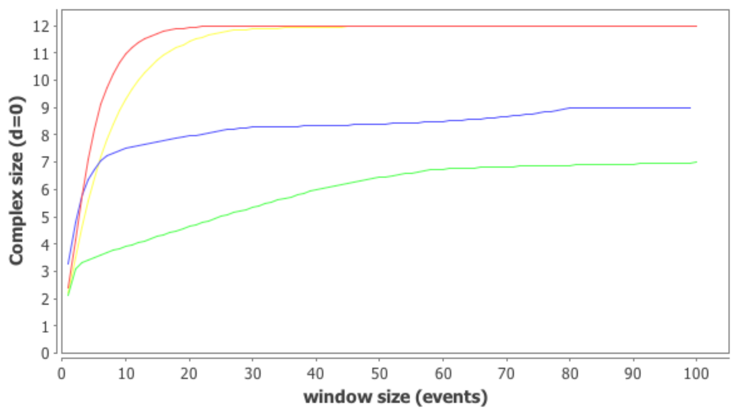
    Mean number of distinct (*n*+1)*-chords played in the same segment*

J. S. Bach – Choral BWV 292

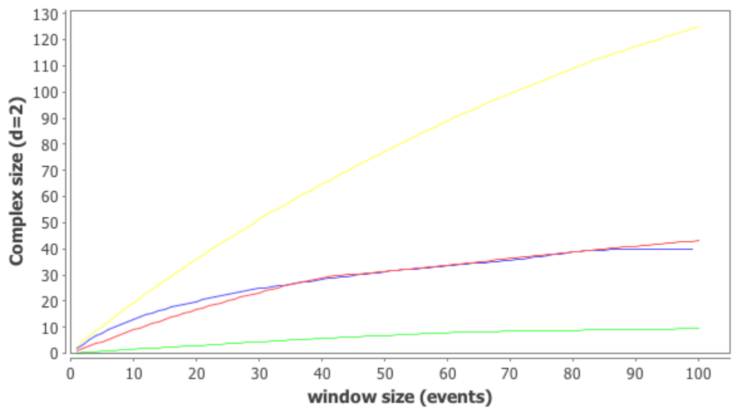
P. Glass – Metamorphosis 1

A. Schoenberg – Op. 33 a

A. Webern – Op. 28 2nd mvt



Size at dimension 0



Size at dimension 2

Piece-based Chord Complexes

- Chord complex built from a piece
- Sequence of complexes resulting from a segmentation
- Meaningful musical properties via topological properties?
  - Dimension
  - Size
- *Perspectives*: other topological properties
  - Betti numbers
  - etc.

Intermediate Summary

- Chord complexes
  - Formalization and generalization of the *Tonnetz*
    - Widely used representation in musical theory and analysis
    - Generic to any system (Z_n) and relation (Z-relation, etc.)
  - Chord catalogues with topological properties
  - Topological analysis of a piece

    Musical interpretations of topological features
- Other contributions
  - Skeleton-based classification
  - Higher-order complexes

    Neighborhood between chord complexes (*e.g.*, voice-leading)

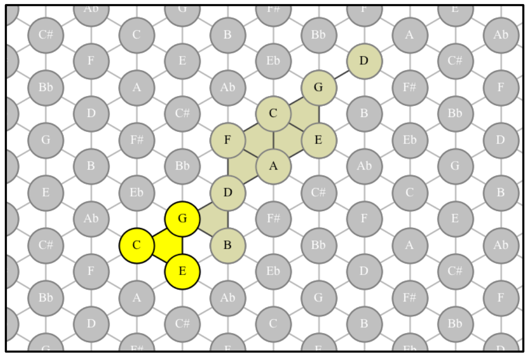
Outline

Bridging the gap between spatial computing and music theory

1. Proof of concept: a spatial study of all-interval series
2. Building chord spaces for music theory and analysis
3. Linking spaces for music generation and analysis
4. Conclusion and perspectives

Motivation

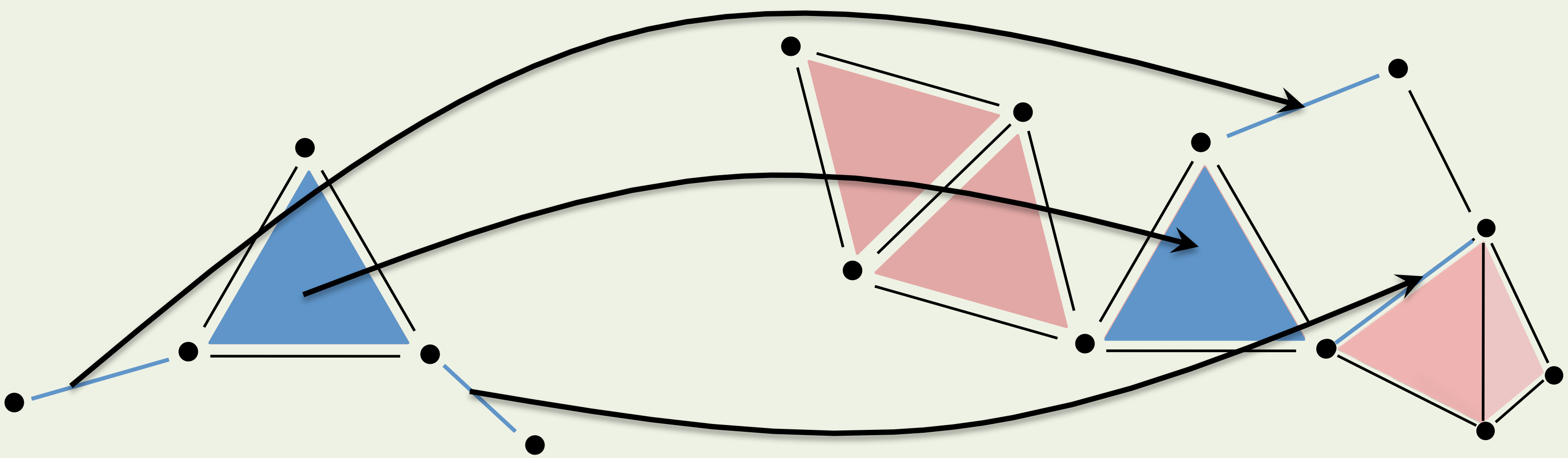
- Representation of movement in chord spaces
  - Musical sequences seen as trajectories in some space
  - Construction of trajectories
- Musical interpretation of spatial transformations
- Analysis of a movement for piece analysis



Linking Chord Complexes

Toolbox: *Morphisms*

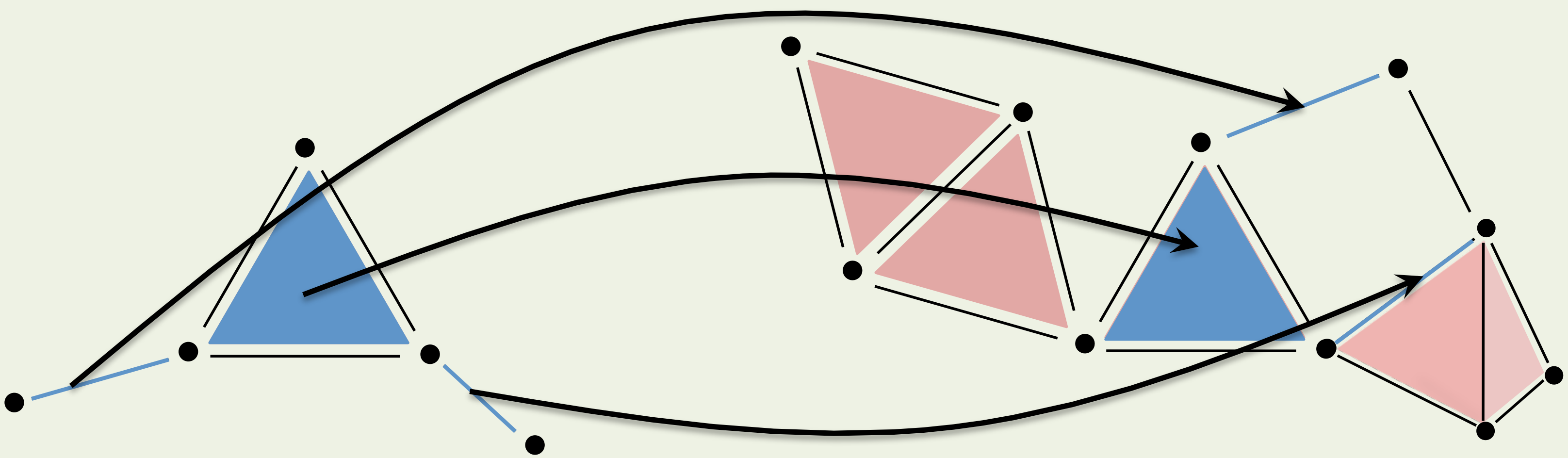
- Function of complexes preserving dimension and incidence
- Extension to topological collections
- *Structural inclusion*: injective morphisms of complexes



Linking Chord Complexes

Toolbox: *Morphisms*

- Function of complexes preserving dimension and incidence
- Extension to topological collections
- *Structural inclusion*: injective morphisms of complexes

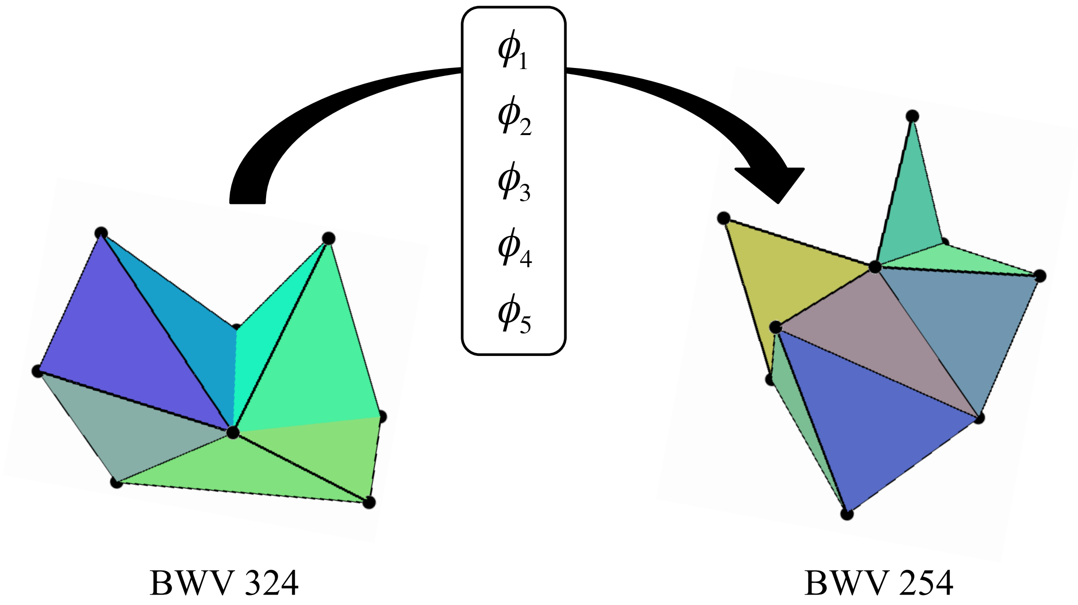


Linking Chord Complexes

- Spatialization of chord sets

  Defining a topological collection to represent a set of chords

- Structural inclusion (≠ *label*)

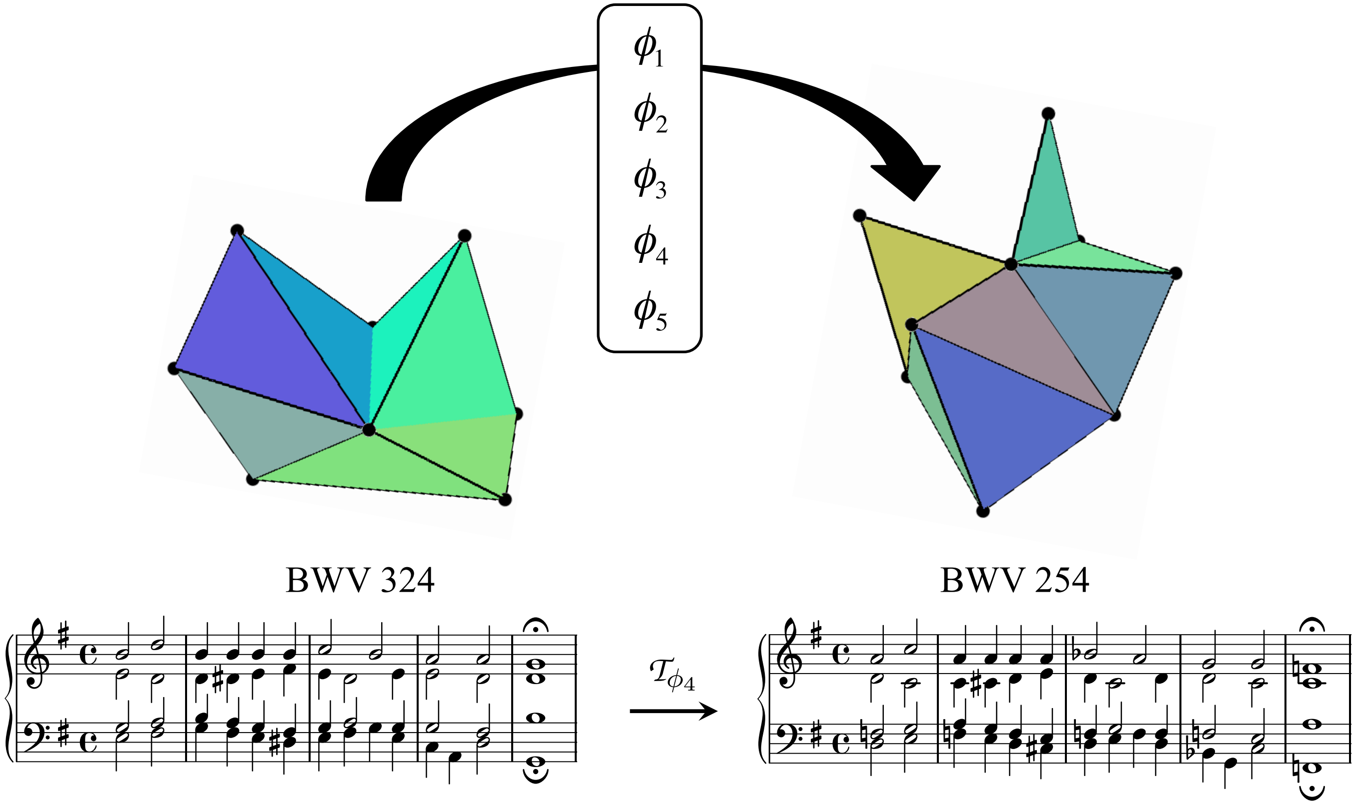


Linking Chord Complexes

- Spatialization of chord sets

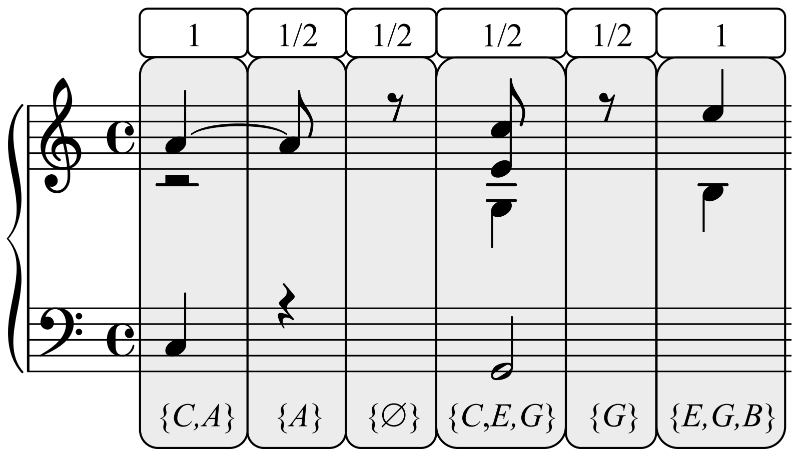
  Defining a topological collection to represent a set of chords

- Structural inclusion (≠ *label*)

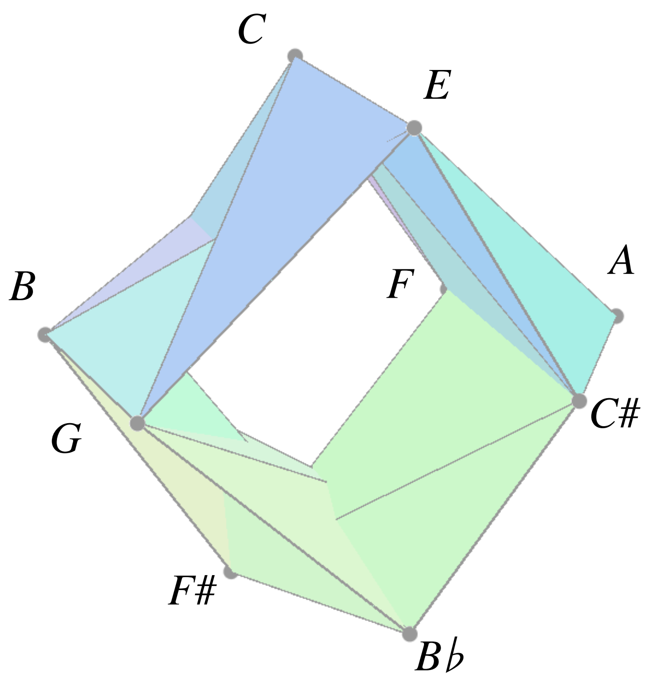


Building Trajectories

- Chord complexes are used as *support spaces*
- Trajectories
  - A segment is represented by a region of the support space



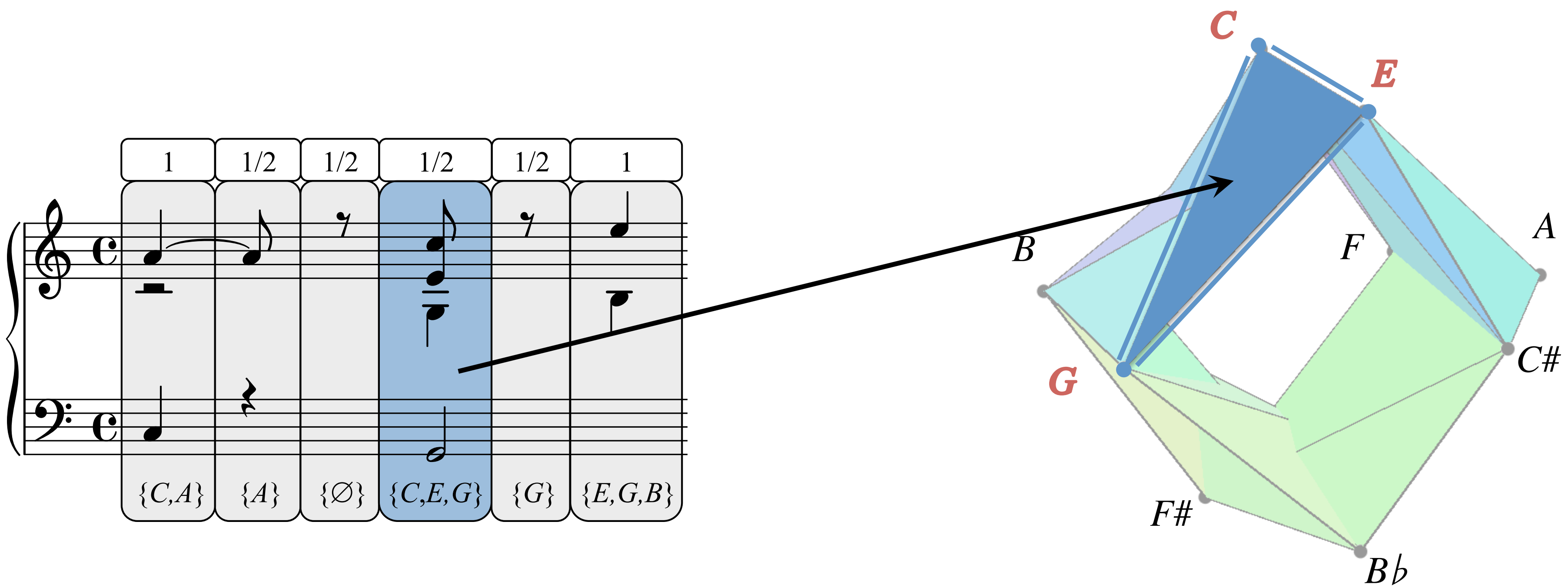
Piece P



Support space $\mathcal{K}$

Building Trajectories

- Chord complexes are used as *support spaces*
- Trajectories
  - A segment is represented by a region of the support space

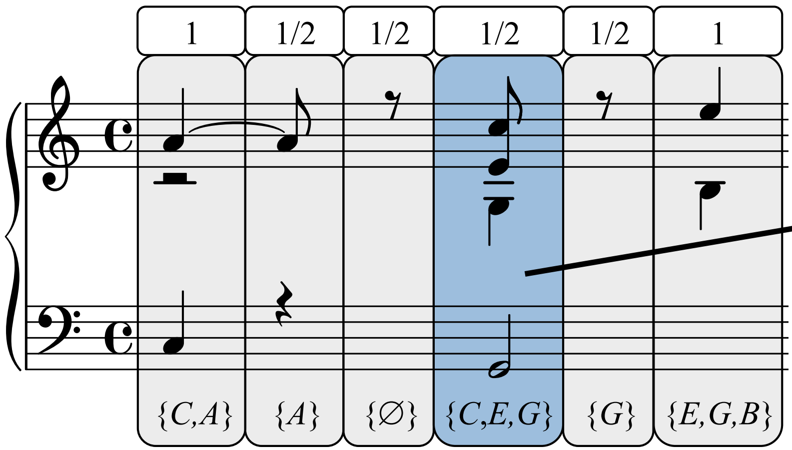


Piece P

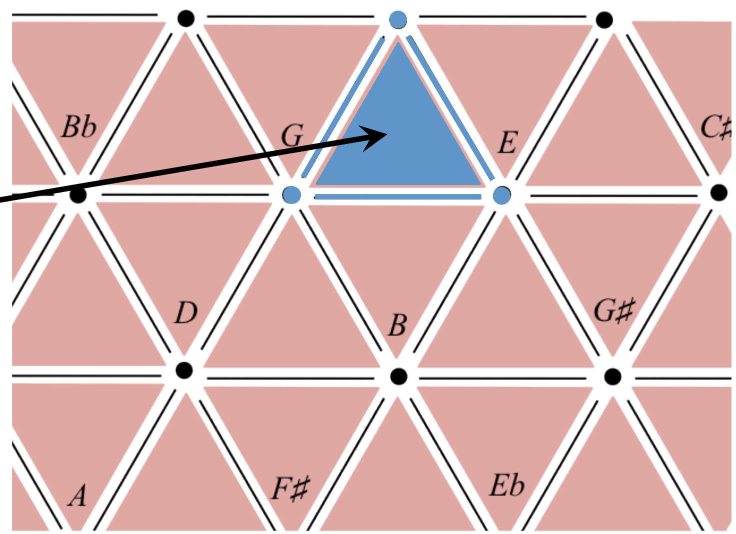
Support space $\mathcal{K}$

Building Trajectories

- Chord complexes are used as *support spaces*
- Trajectories
  - A segment is represented by a region of the support space



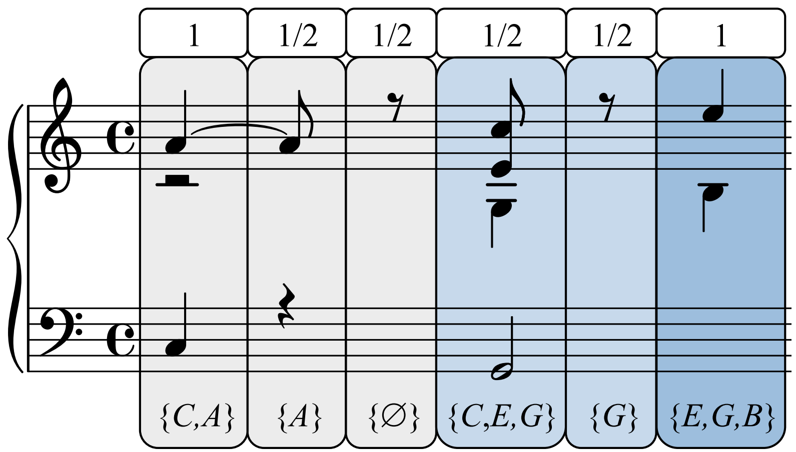
Piece P



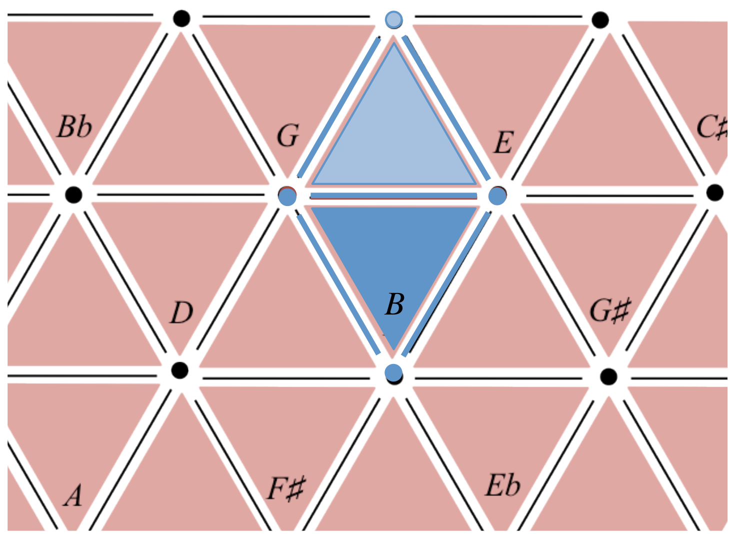
Unfolded support space $\mathcal{K}^{\mathcal{U}}$

Building Trajectories

- Chord complexes are used as *support spaces*
- Trajectories
  - A segment is represented by a region of the support space
  - Chords can be represented in multiple locations



Piece P

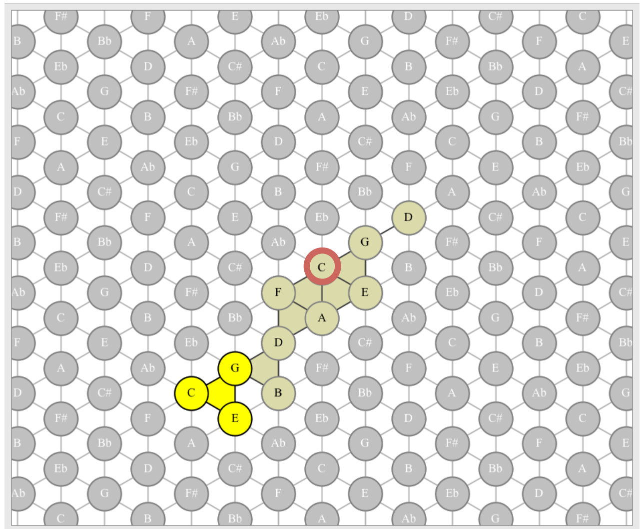


Unfolded support space $\mathcal{K}^{\mathcal{U}}$

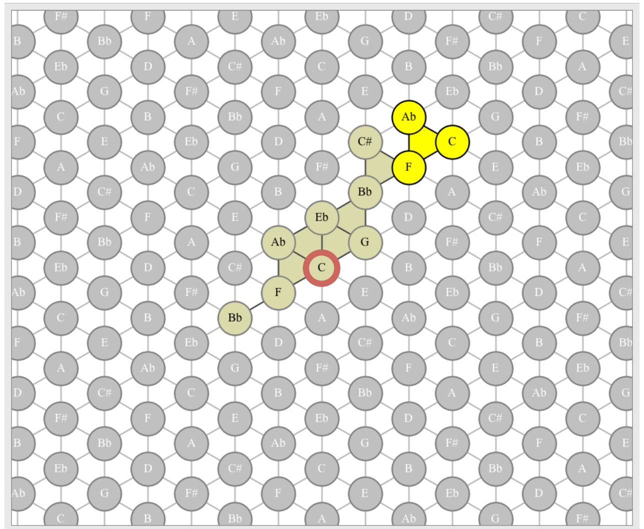
Trajectory Transformations

- Automorphism of the support space

  Geometrical transformations of trajectories



point reflection →

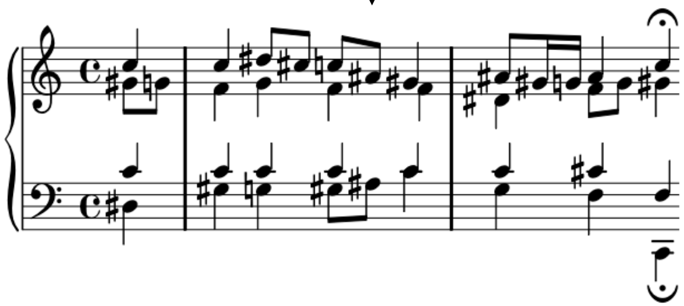


$\mathcal{K}^{u}{}_{TI}[3,4,5]$ ↑

$\mathcal{K}^{u}{}_{TI}[3,4,5]$ ↓

J.-S.Bach - Choral BWV 256

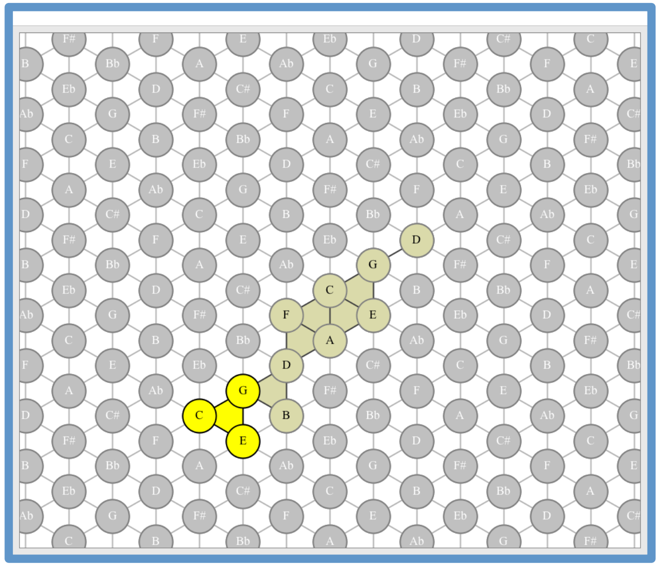
→ *pitch inversion*

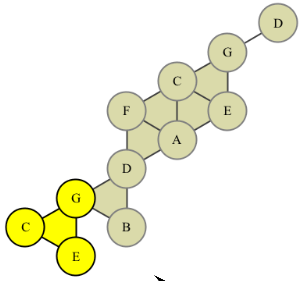


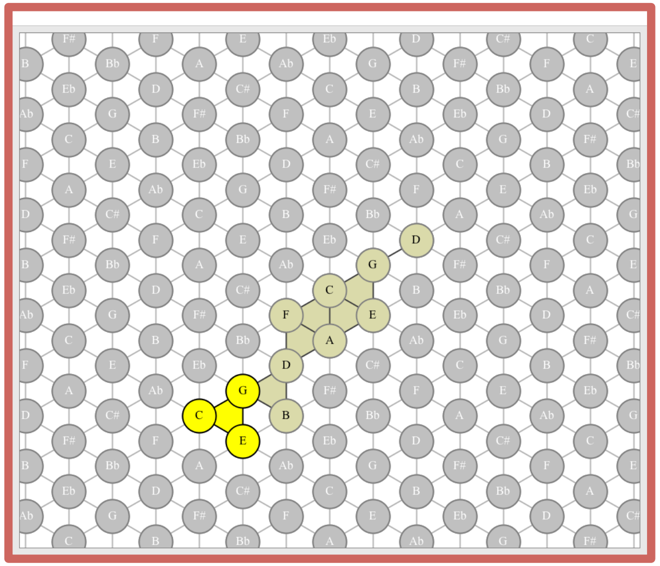
Trajectory Transformations

- Isomorphism from a support space to another

  Transformation of the initial space of the trajectory







$\mathcal{K}^{u}_{TI}[3,4,5]$

$\mathcal{K}^{u}_{TI}[2,3,7]$

?

J.-S.Bach - Choral BWV 256

Trajectory Transformations

- Musical interpretation of spatial transformations

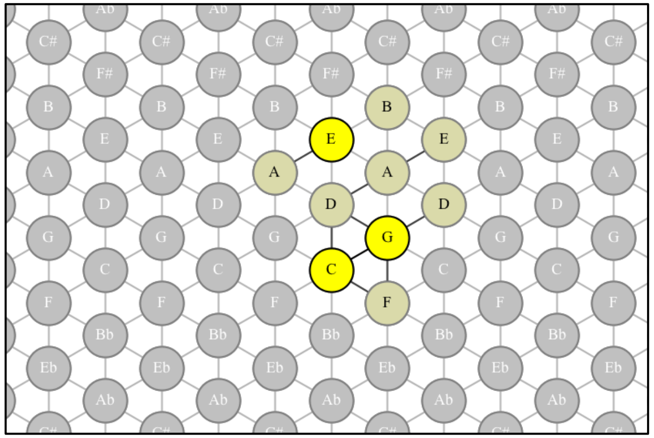
Transformation sur l'espace	Transformation sur la trajectoire	Transformation musicale	Régularité
$\mathcal{K}^{u}{}_{\mathcal{TT}}[X_{chro}] \rightarrow \mathcal{K}^{u}{}_{\mathcal{TT}}[X_{chro}]$	Translation	Transposition chromatique	Régulier
	Symétrie centrale	Inversion chromatique	Régulier
	Rotation d'angle $\neq\pi$ Symétrie axiale	?	Semi-régulier
	Homothétie ($\Leftrightarrow \mathcal{K}^{u}{}_{\mathcal{TT}}[X_{chro}] \rightarrow \mathcal{K}^{u}{}_{\mathcal{TT}}[X'_{chro}]$)	?	Semi-régulier
$\mathcal{K}^{u}{}_{\mathcal{TT}}[X_{hep}]_{T} \rightarrow \mathcal{K}^{u}{}_{\mathcal{TT}}[X_{hep}]_{T}$	Translation	Transposition modale	Régulier
	Symétrie centrale	Inversion modale	Régulier
	Rotation d'angle $\neq\pi$ Symétrie axiale	?	Semi-régulier
	Homothétie ($\Leftrightarrow \mathcal{K}^{u}{}_{\mathcal{TT}}[X_{hep}]_{T} \rightarrow \mathcal{K}^{u}{}_{\mathcal{TT}}[X'_{hep}]_{T}$)	?	Semi-régulier
$\mathcal{K}^{u}{}_{\mathcal{TT}}[X_{chro}] \rightarrow \mathcal{K}^{u}{}_{\mathcal{TT}}[X'_{chro}]$	Plongement	?	Semi-régulier
$\mathcal{K}^{u}{}_{\mathcal{TT}}[X_{hep}]_{T} \rightarrow \mathcal{K}^{u}{}_{\mathcal{TT}}[X'_{hep}]_{T}$	Plongement	?	Semi-régulier
$\mathcal{K}^{u}{}_{\mathcal{TT}}[X_{hep}]_{T} \rightarrow \mathcal{K}^{u}{}_{\mathcal{TT}}[X_{hep}]_{T'}$	Plongement	Transposition chromatique (+ transposition modale)	Régulier
Trace $\rightarrow$ Trace	Isométrie	Permutation dans le temps des ensembles de notes	Irrégulier
$\mathcal{K} \rightarrow \mathcal{K}$	Isométrie	?	Irrégulier
$\mathcal{K} \rightarrow \mathcal{K}'$	Plongement	?	Irrégulier

Trajectory Analysis

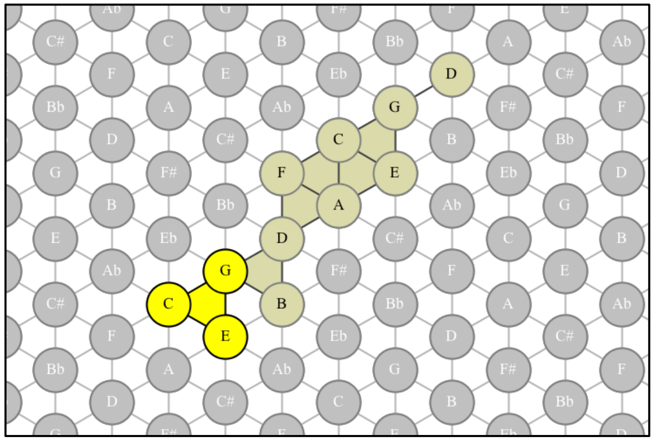
- The aspect of a trajectory depends on the support space

J.-S.Bach - Choral BWV 256

→



$\mathcal{K}_{\mathrm{TI}}[2,5,5]$



$\mathcal{K}_{\mathrm{TI}}[3,4,5]$

Trajectory Analysis

- The "look" of a trajectory depends on the support space
- Notion of *compliance*
  - Propension of a space to fit with a piece
  - Computation based on compactness
    - " *The more compact is a trajectory,*

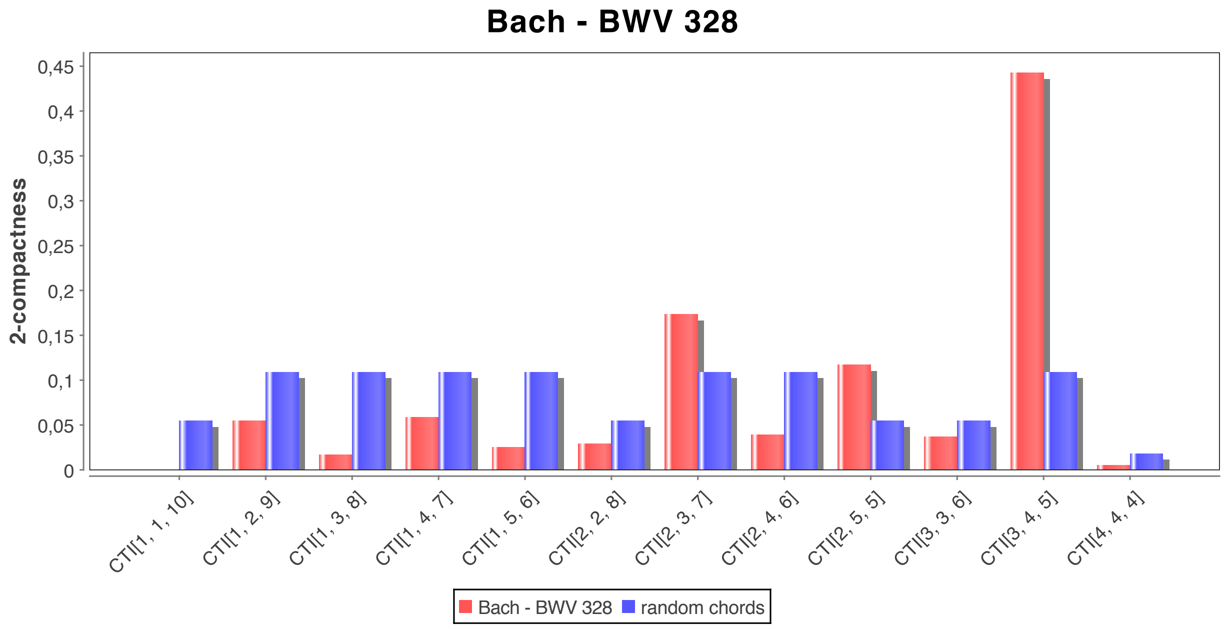
      the more compliant is the support space with the piece "
    - Compactness at a dimension d

      Simplicial collection A, complex $\mathcal{K}$, injective morphism τ from A to $\mathcal{K}$

$$\max_{\tau} \frac{f_{d+1}(\tau(A))}{f_{d+1}(A)} = \max_{\tau} \frac{f_{d+1}(\tau(A))}{\binom{f_1(A)}{d+1}} \quad \text{if } A \text{ is a simplex}$$

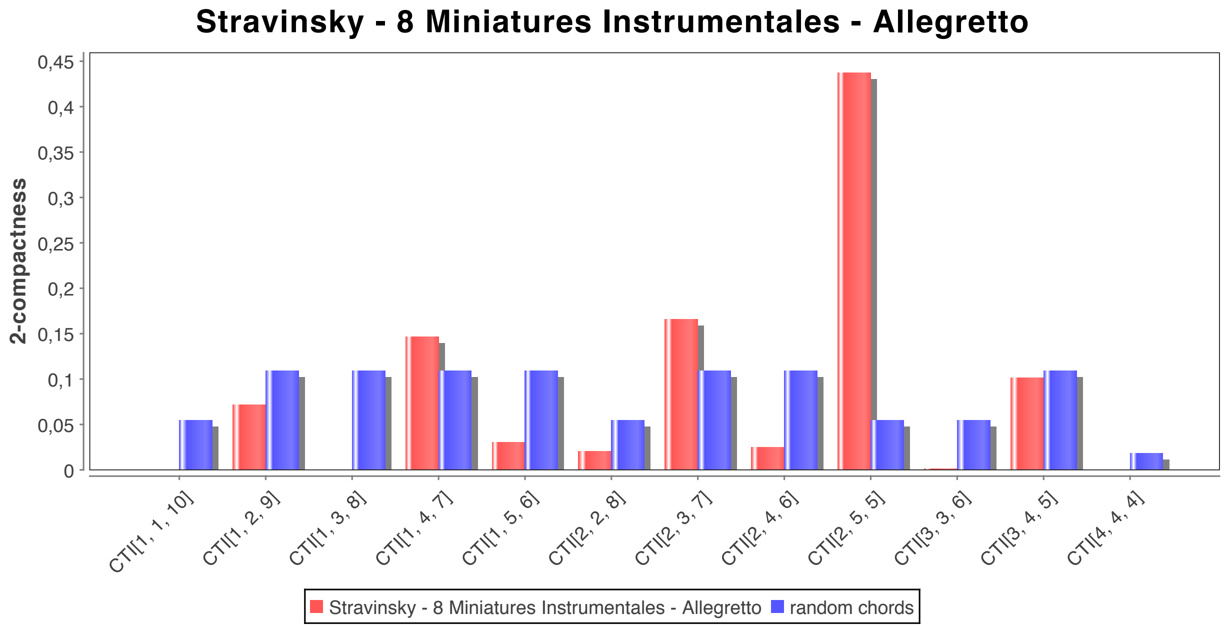
Trajectory Analysis

- Complex compliance: some examples



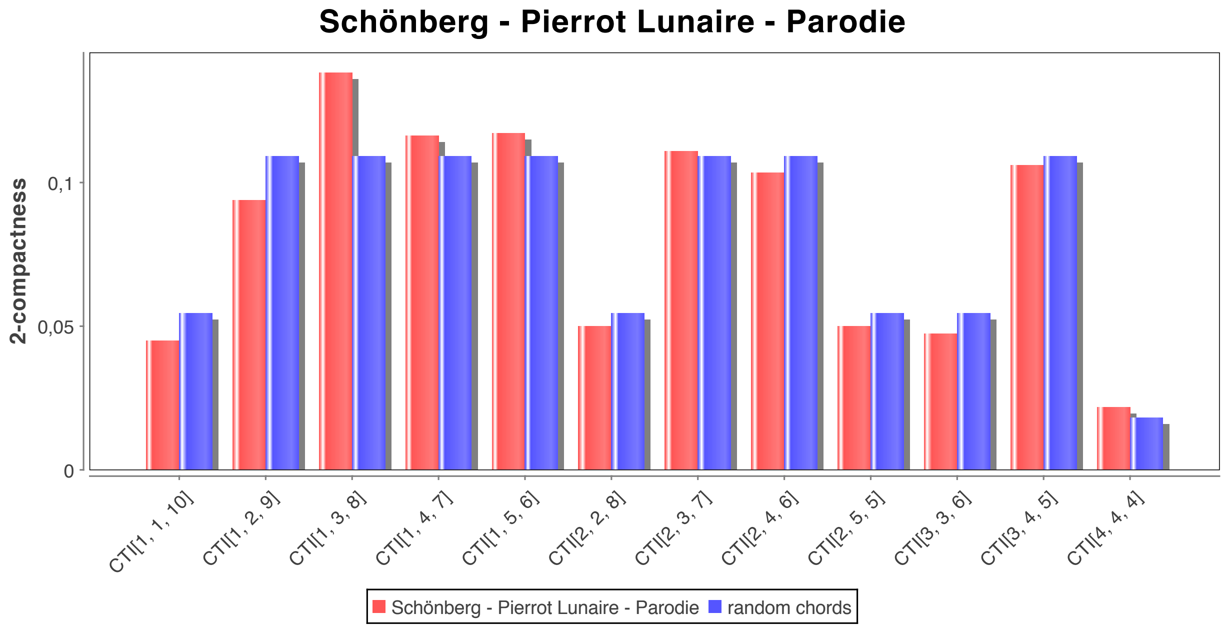
Trajectory Analysis

- Complex compliance: some examples



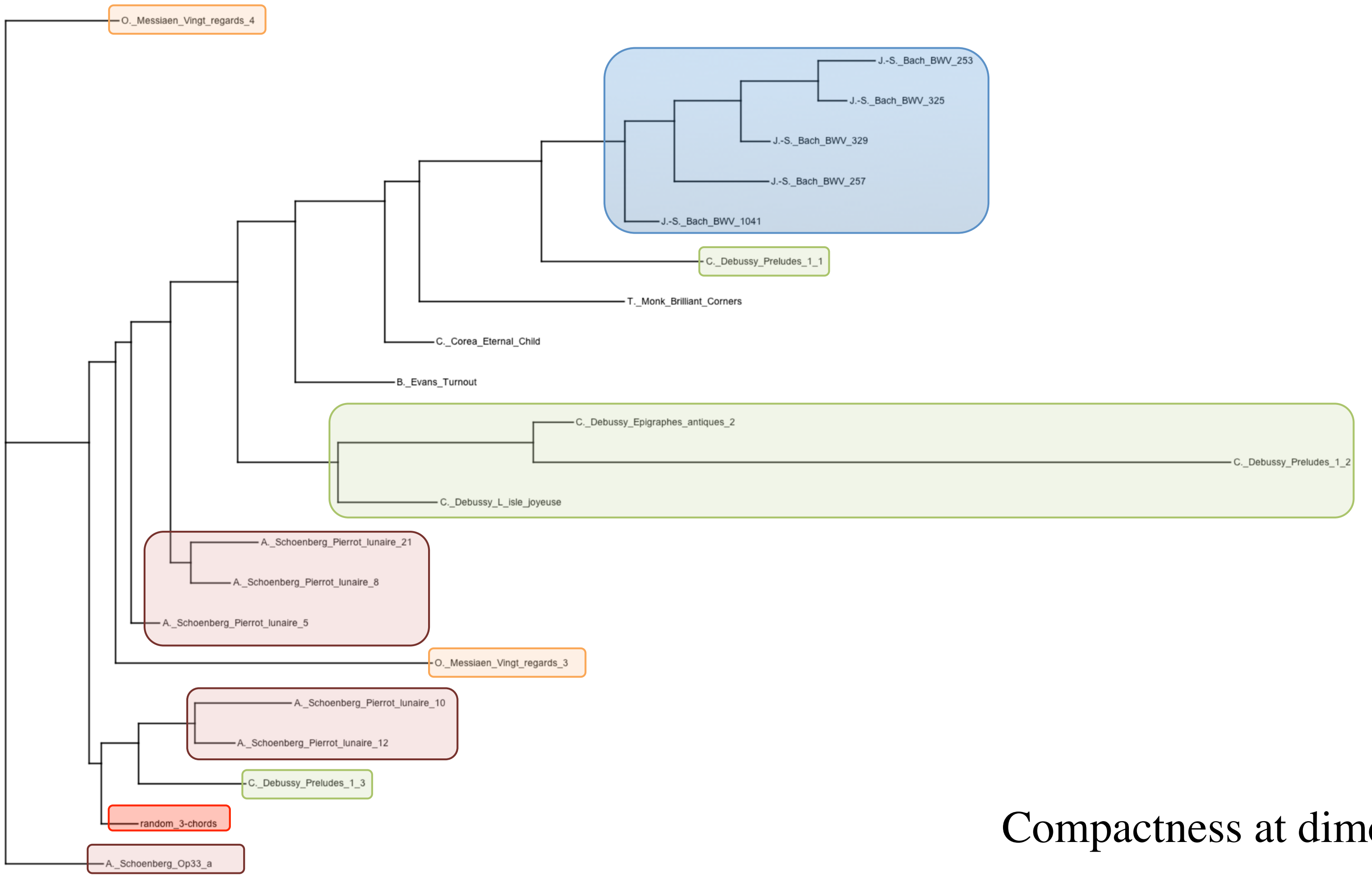
Trajectory Analysis

- Complex compliance: some examples



Trajectory Analysis

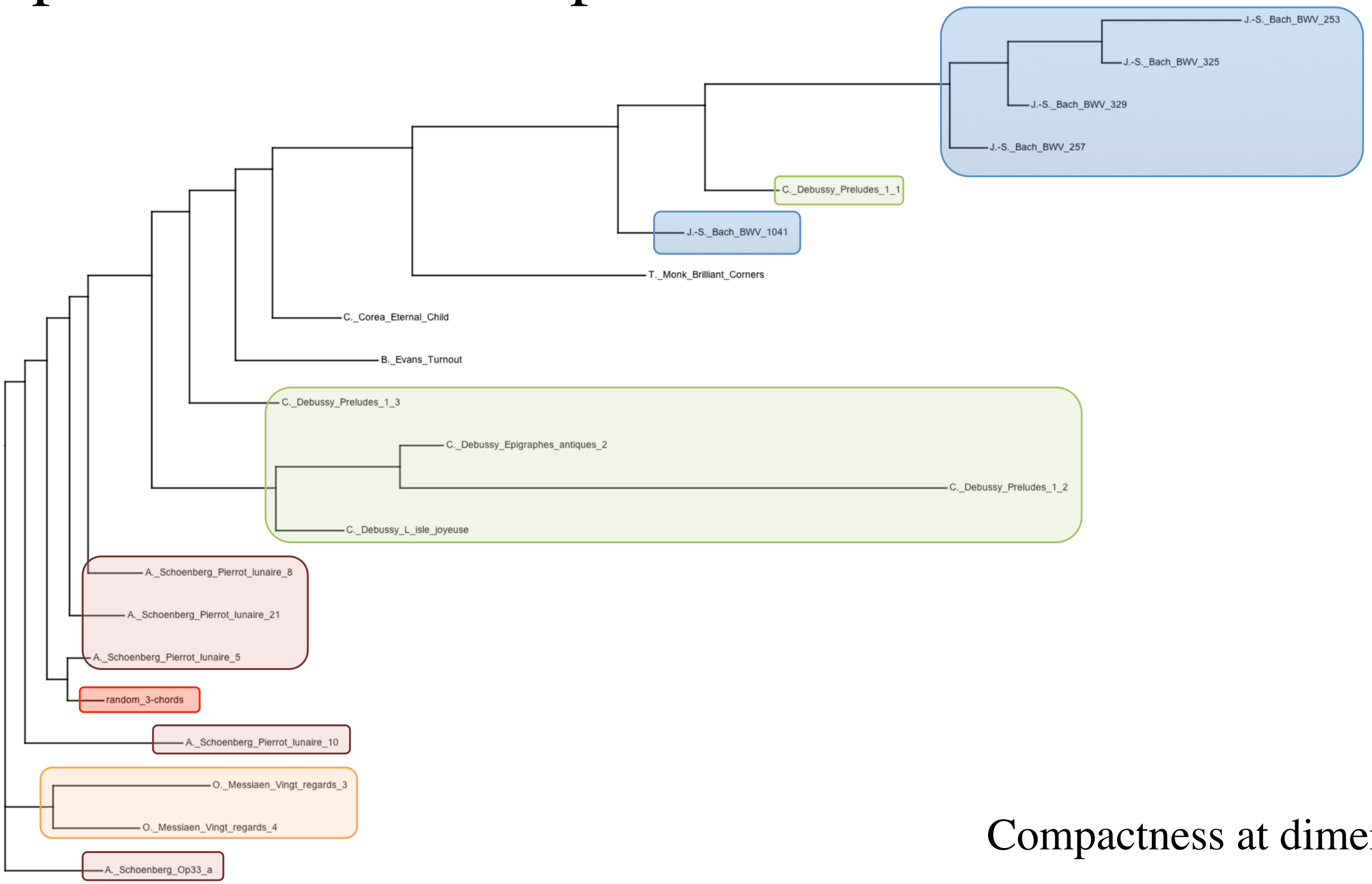
- Complex compliance: some examples
- Compliance of a set of spaces for music classification



Compactness at dimension 1

Trajectory Analysis

- Complex compliance: some examples
- Compliance of a set of spaces for music classification



Compactness at dimension 2

Intermediate Summary

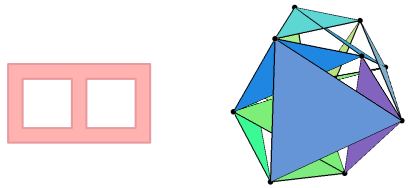
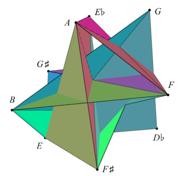
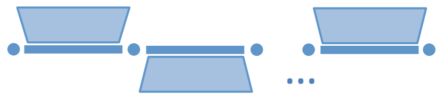
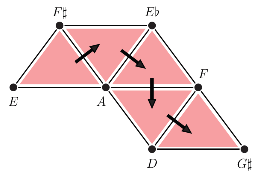
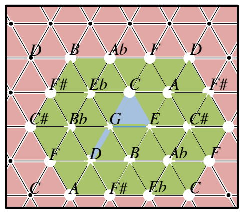
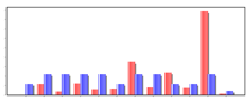
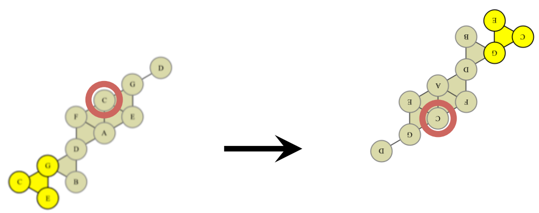
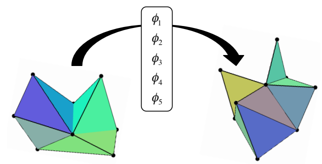
- Trajectories enable to formalize the use chord-class complexes (including *Tonnetz*) for musical analysis
- Spatial representation of a system evolving in time
  - Spatial transformations
    - Spatial transformations relate to familiar or new musical operations
    - Spatial transformations only affect pitches (time is not structurally represented)
  - Spatial analysis (for example: compactness computation)
    - Compliant underlying spaces can be viewed as a signature of a style
    - From static compliance to dynamic compliance

Outline

Bridging the gap between spatial computing and music theory

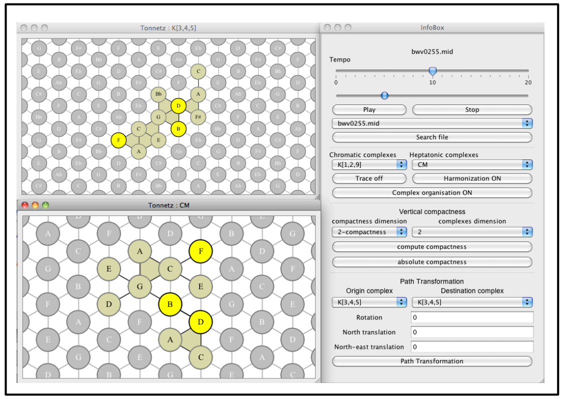
1. Proof of concept: a spatial study of all-interval series
2. Building chord spaces for music theory and analysis
3. Linking spaces for music generation and analysis
4. Conclusion and perspectives

Summary

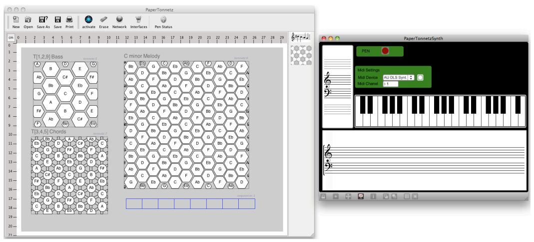
	Algebraic complex	Piece complex
Construction		
Trajectory generation		
Piece representation	 	
Piece transformation		

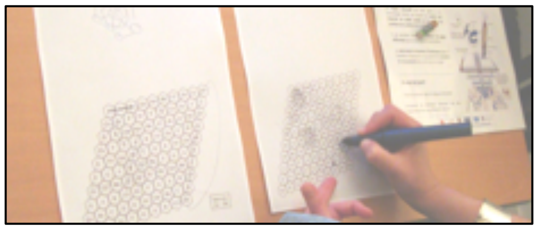
Summary

- Tool development
  - HexaChord
    - Automatic construction of trajectories in chord complexes
    - Compliance computation
    - Transformations on MIDI files



  - PaperTonnetz (with J. Garcia/LRI-IRCAM)
    - Composition in chord complexes with interactive paper



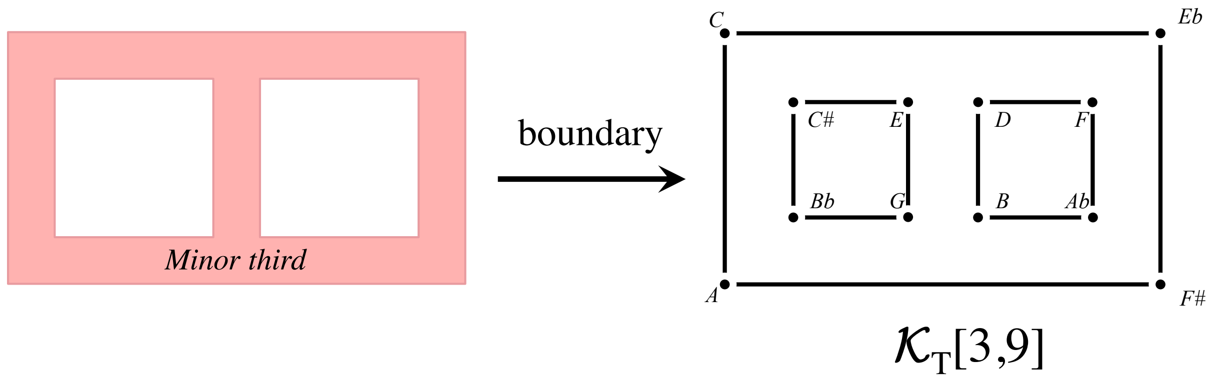


Benefits of the spatial representation

- Understanding
  - Intuitive visualization of regularities in musical patterns
- Technical
  - Use of spatial tools for musical problems
- Heuristic
  - Original representations inspire new approaches:
    - in music formalization
    - in music analysis
    - in music generation

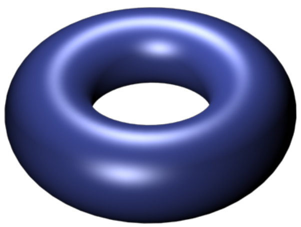
Perspectives

- Complex construction:
  - Extension to other equivalence classes
    - other scales (Z_5, Z_8, $Z_{n>12,...}$)
    - other equivalence relations (Z-relation,…)
  - Combinatorial representation of other musical features (timbres,…)
  - Different abstraction levels for topological studies
  - A unified viewpoint with the AIS spaces
    - A chord complex can be viewed as the boundary of a higher dimensional cell representing the chord class



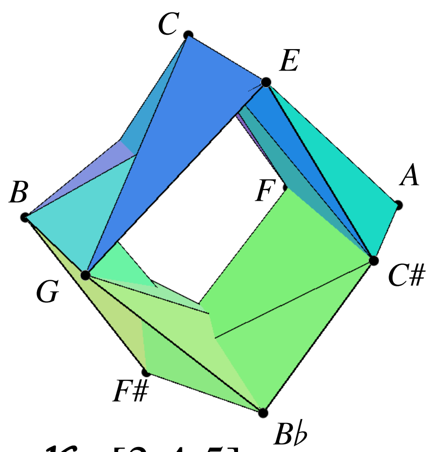
Perspectives

- Complex construction:
  - Extension to other equivalence classes
    - other scales ($Z_5, Z_8, Z_{n>12,\ldots}$)
    - other equivalence relations (Z-relation,…)
  - Combinatorial representation of other musical features (timbres,…)
  - Different abstraction levels for topological studies
  - A unified viewpoint with the AIS spaces
    - A chord complex can be viewed as the boundary of a higher dimensional cell representing the chord class



[3,4,5] *chord class*

boundary



$\mathcal{K}_{TI}[3,4,5]$

Perspectives

- Trajectories
  - Transformations
    - Spatial composition based on *new* transformations
    - Transformations of other musical aspects (rhythm, etc.)
  - Compliance
    - Compactness of *transitions* between chords
    - Evaluation of other spatial features (periodicity of the movement, etc.)
  - Algorithmic composition
    - Trajectory generation in chord class complexes
- HexaChord:
  - Complete availability for music theorists
  - Integration of audio inputs

Scientific contributions

- **Journal**
  - L. Bigo, A. Spicher. *Self-Assembly of Musical Representations in MGS*.
    International Journal of Unconventionnal Computing. Accepted for publication
- **Conferences**
  - L. Bigo, A. Spicher and O. Michel. *Spatial Programming for Music Representation and Analysis*.
    Spatial Computing Workshop 2010.
  - L. Bigo, A. Spicher and O. Michel. *Two Representations of Music Computed with a Spatial Programming Language*
    New Worlds of Computation 2011.
  - L. Bigo, J-L. Giavitto and A. Spicher. *Building Topological Spaces for Musical Objects*
    Mathematics and Computation in Music 2011.
  - L. Bigo, J. Garcia, A. Spicher and W. E. Mackay. *PaperTonnetz : Music Composition with Interactive Paper.*
    Sound and Music Computing 2012.
  - L. Bigo and A. Spicher. *Self-Assembly of Musical Representations in MGS*.
    Artificial Intelligence and Simulation of Behaviour Convention 2013.
  - L. Bigo, J-L. Giavitto and A. Spicher. *Spatial Programming for Musical Transformations and Harmonization*.
    Spatial Computing Workshop 2013.
  - L. Bigo, M. Andreatta, J-L. Giavitto and A. Spicher. *Computation and Visualization of Musical Structures in Chord-based Simplicial Complexes*. Mathematics and Computation in Music 2013.
  - J. Garcia, L. Bigo, A. Spicher and W E. Mackay. *PaperTonnetz : Supporting Music Composition with Interactive Paper*
    ACM SIGCHI Conference on Human Factors in Computing Systems 2013.
- **Talks**
  - LACL, LRI, MaMuX seminar, GRATOS, Queen Mary University, LaBRI, Lip6, McGill University, Journées science et musique
- **Award**
  - Prize young researcher 2013 in science and music (IRISA, AFIM)
- **Softwares**
  - HexaChord
  - PaperTonnetz (with Jérémie Garcia/LRI-IRCAM)