MGS

a DSL for modeling and simulating $(DS)^2$

Introduction

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www.spatial-computing.org/mgs/iccsa14

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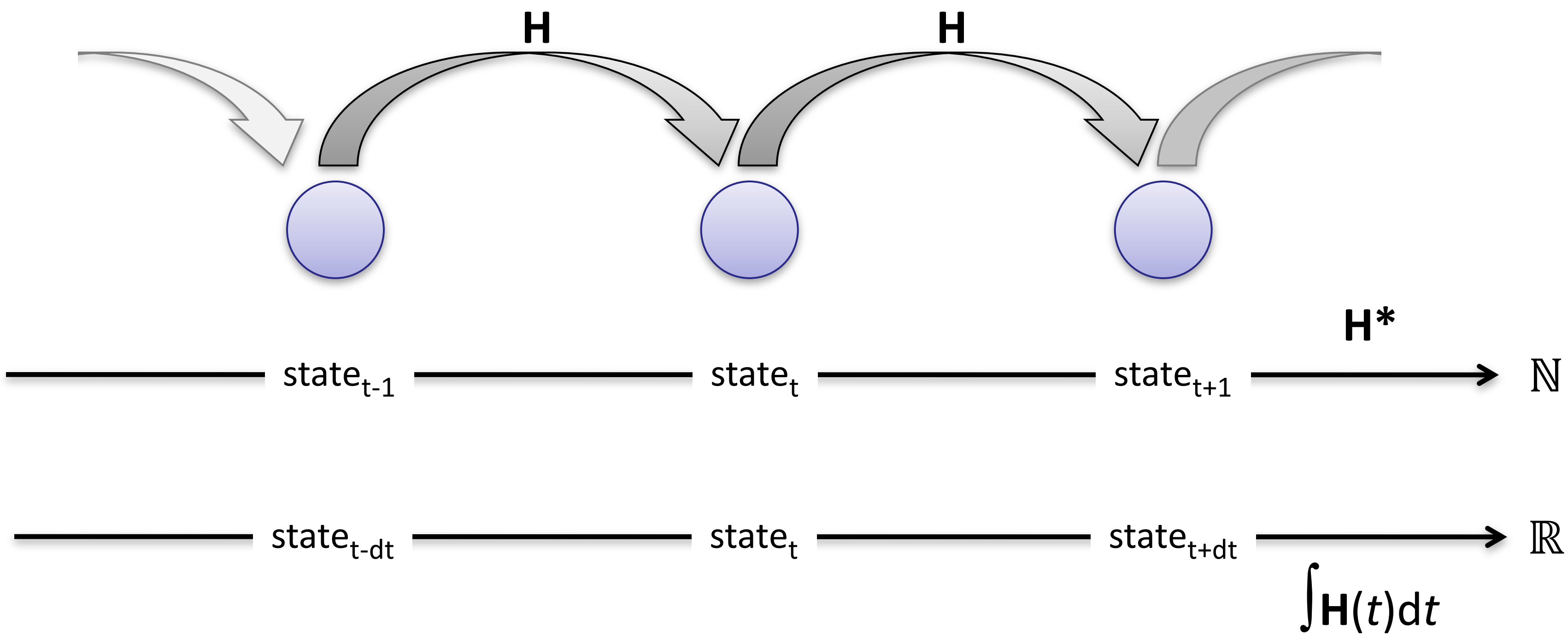
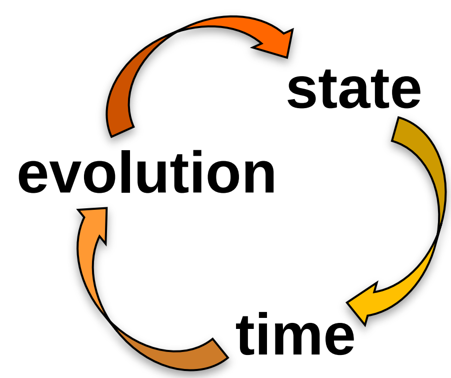
Simulation of dynamical system…



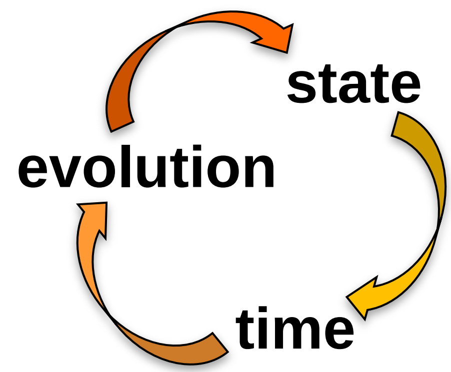
Specification of

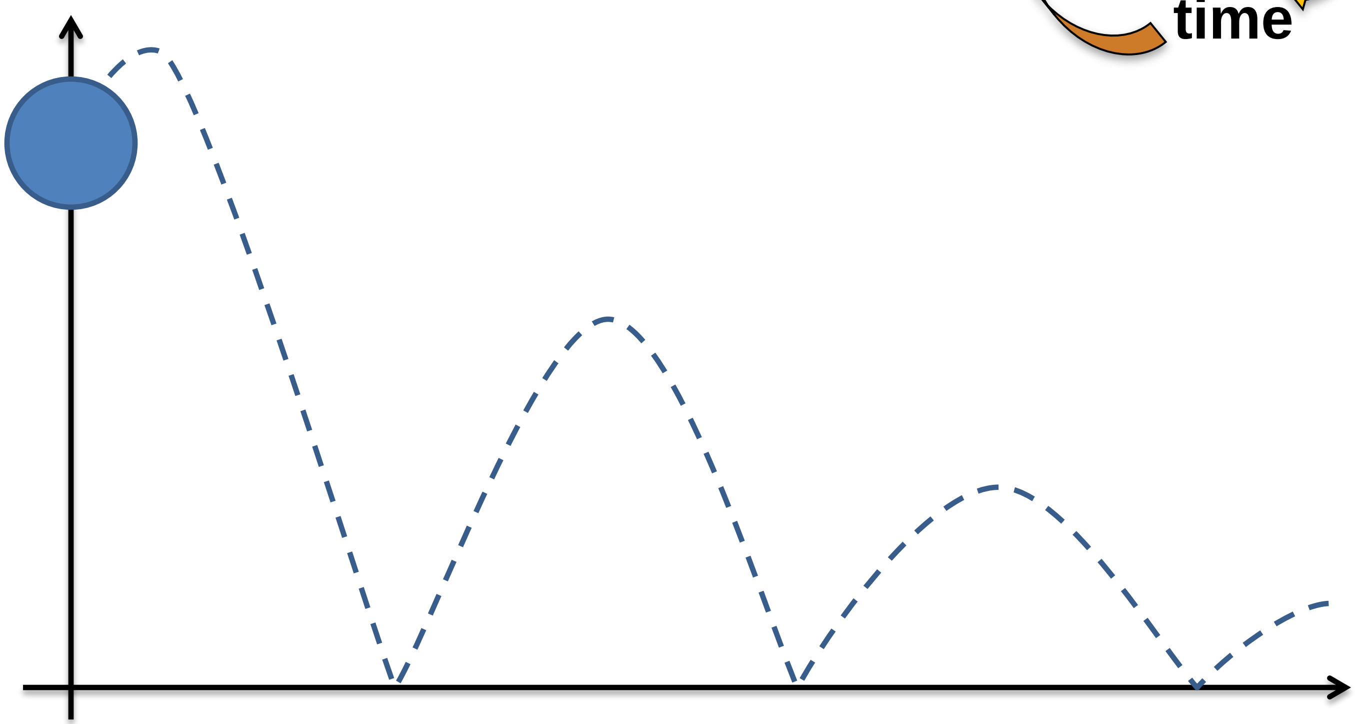
- **the state structure**
- **the model of time**
- **the evolution function**



Simulation of dynamical system…

- Example: bouncing ball
  - State: *position* $\mathbf{p}$ & *speed* $\vec{v}$
  - Time: *continuous*
  - Evolution function: *gravity* & *collision*





Simulation of dynamical system…

- What about complex systems?
  - Falling ball

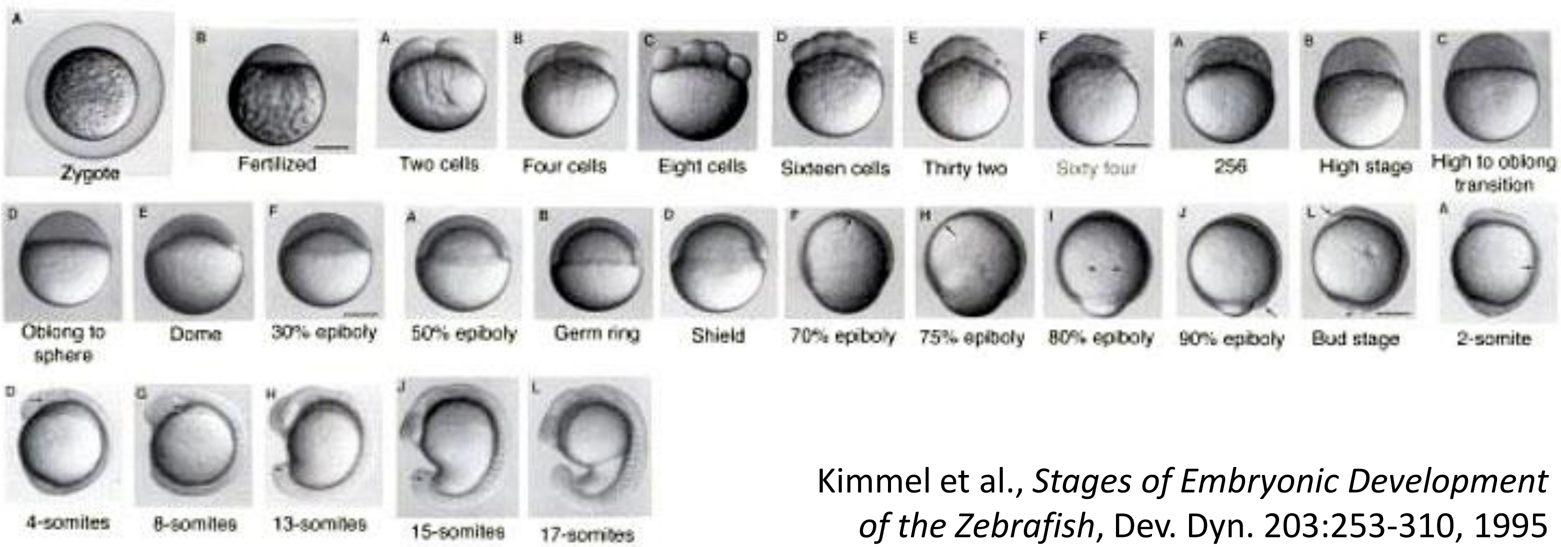
    At **any** time, the state is defined by ***exactly two*** vectors (position & speed)

…with a dynamical structure

- What about complex systems?
  - Falling ball

    At **any** time, the state is defined by ***exactly two*** vectors (position & speed)
  - Developing embryo

    At a **given** time, the state is defined
    - A variable number of cells (geometry, concentration, …)
    - A variable organization (division, migration, apoptosis, …)



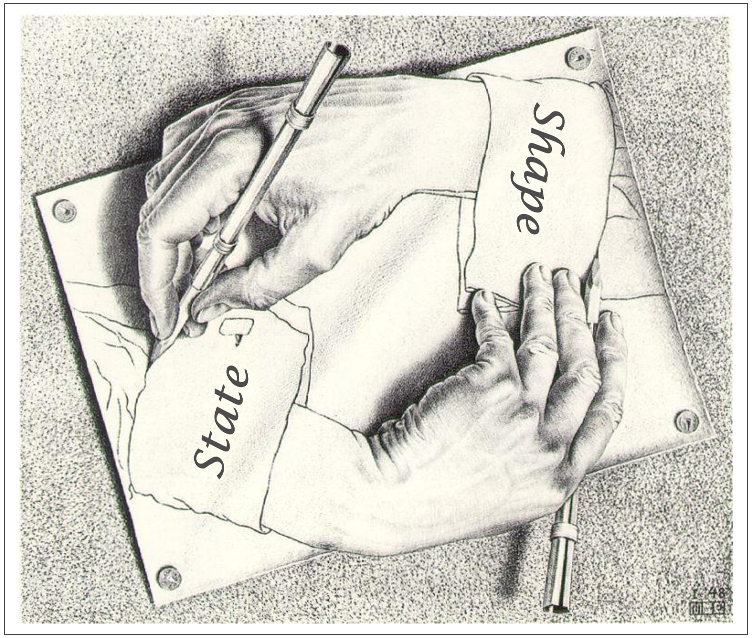
Kimmel et al., *Stages of Embryonic Development of the Zebrafish*, Dev. Dyn. 203:253-310, 1995

…with a dynamical structure

- *Dynamical Systems with a Dynamical Structure*

  The structure of the system evolves jointly with the system

  - *The structure constraints the system evolution which modifies the structure*
  - State space cannot be defined *a priori*



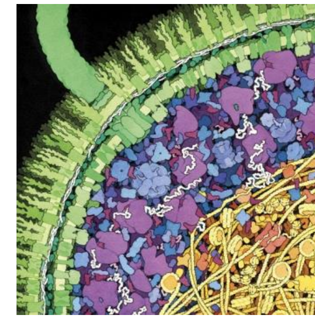
Dynamics
OF the shape

Dynamics
ON the shape

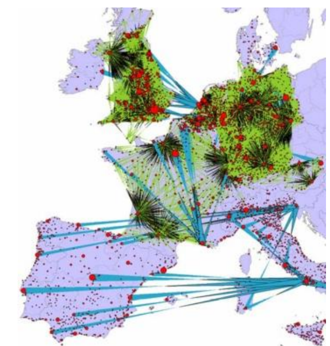
…with a dynamical structure

- Examples of $(DS)^2$
  - In biology
    - Molecular bio., developmental bio.
  - In physics
    - Soft matter mechanics, multi-scale systems
    - General relativity
  - In SHS
    - Urbanism, traffic control
    - Economics
  - In computer science
    - Internet, social network
    - Reconfigurable robots

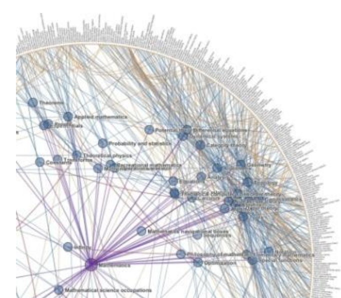
P. Prusinkiewicz



D. Goodsell



M. Satoshi



Outline

- Introduction to MGS
  - Interaction-based modeling
  - Presentation of MGS
- Demonstrations
  - Lindemayer Systems
  - Chemical-like Systems
  - Cellular Automata
  - Multi-agent Systems

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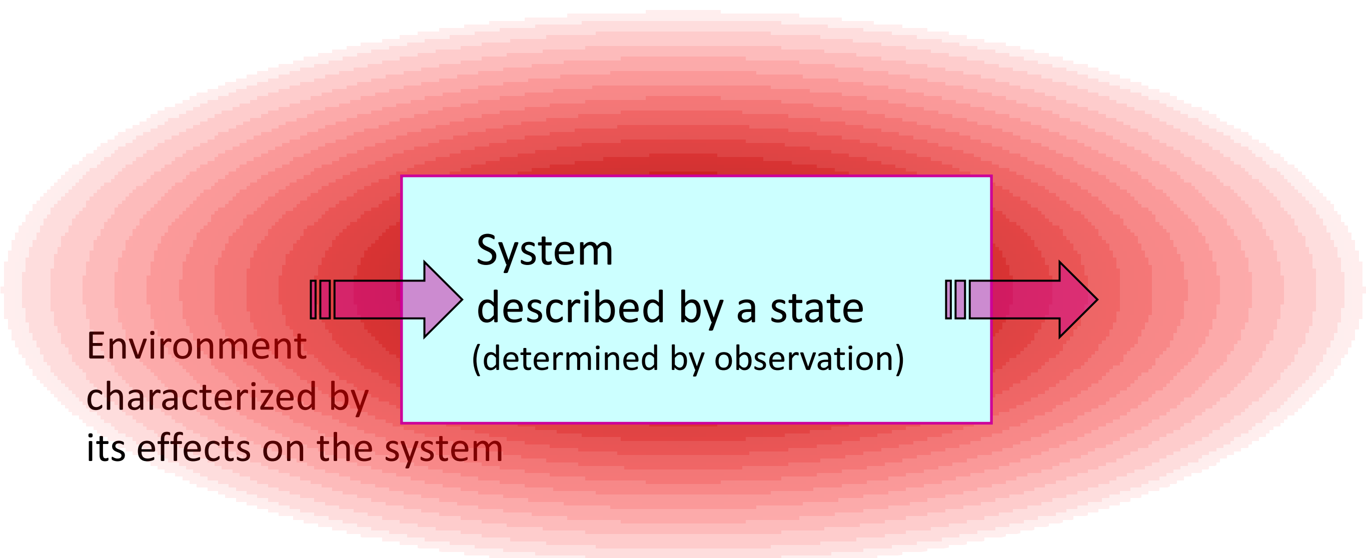
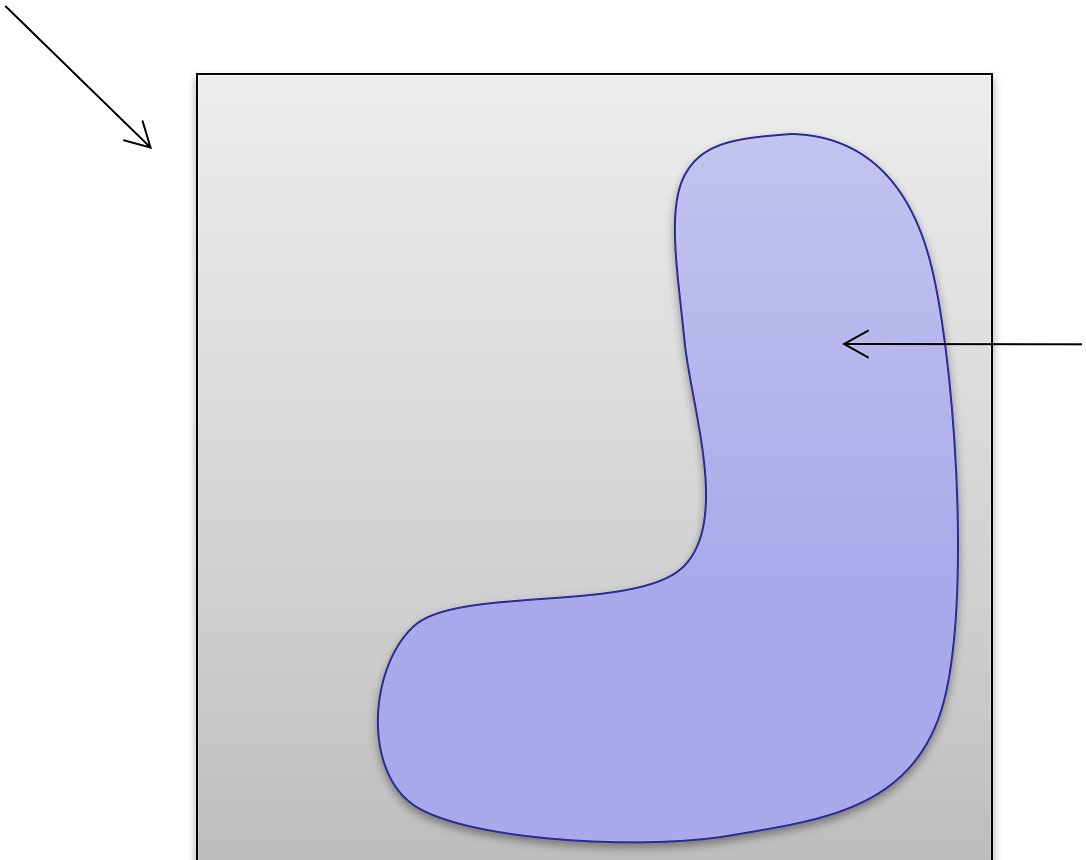
Topology of Interactions

- Let's observe the system
  - State of the system given by observation
  - Structure is dynamic ⇒ structure is an *observable*



Topology of Interactions

A system in some state

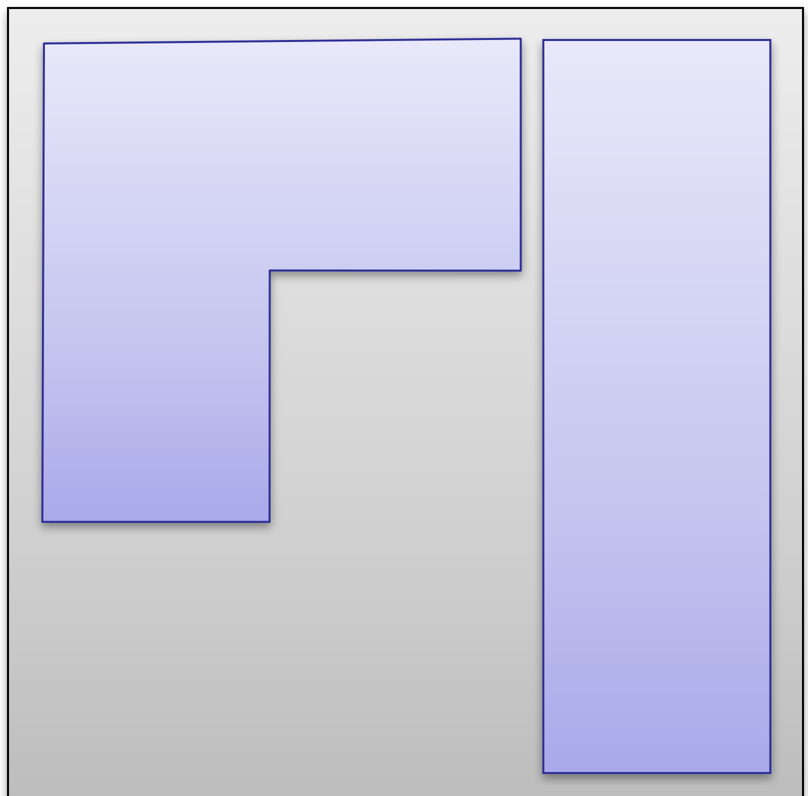


Part of a system that evolves.

Can be identified by comparison with the previous global state

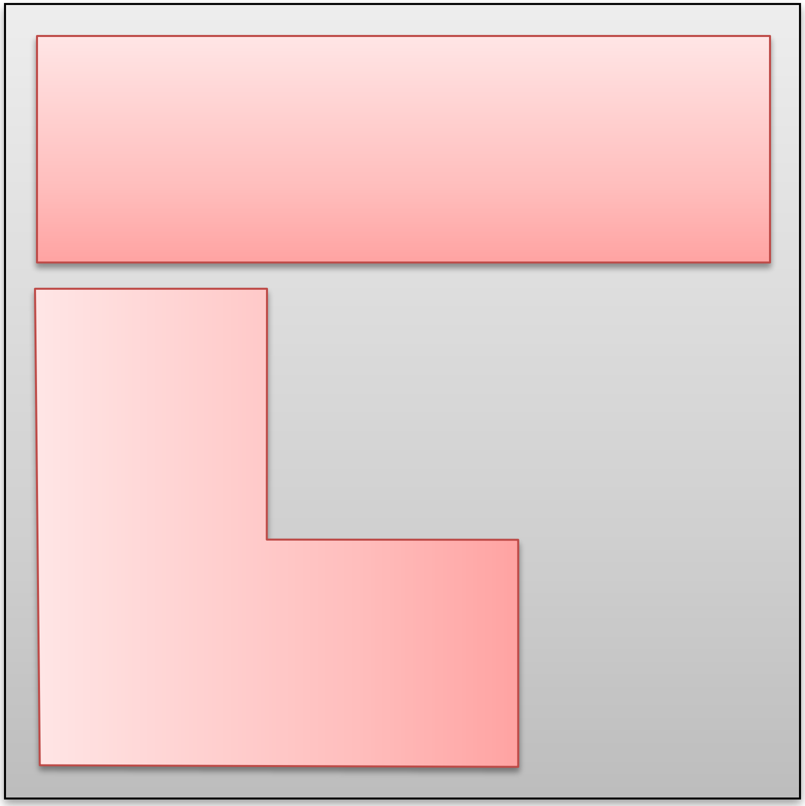
Topology of Interactions

$t = 1$



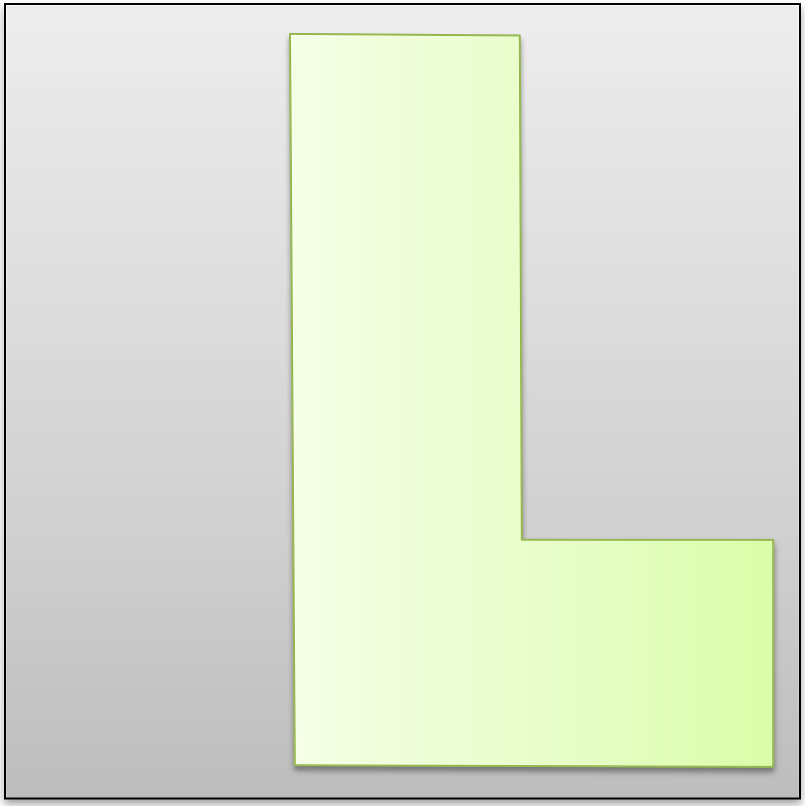
Topology of Interactions

t = 2



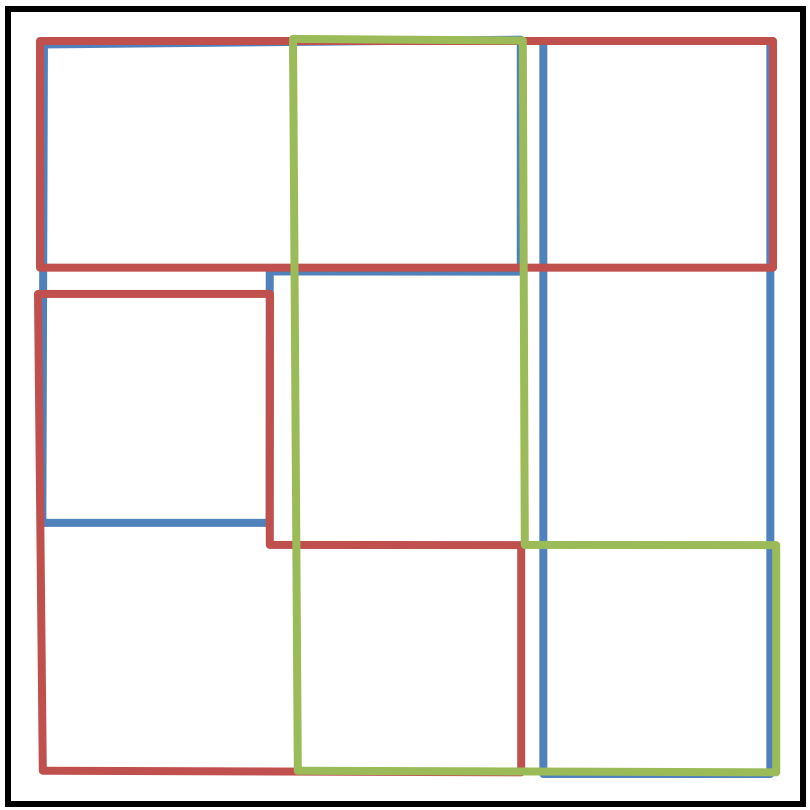
Topology of Interactions

$t = 3$



Topology of Interactions

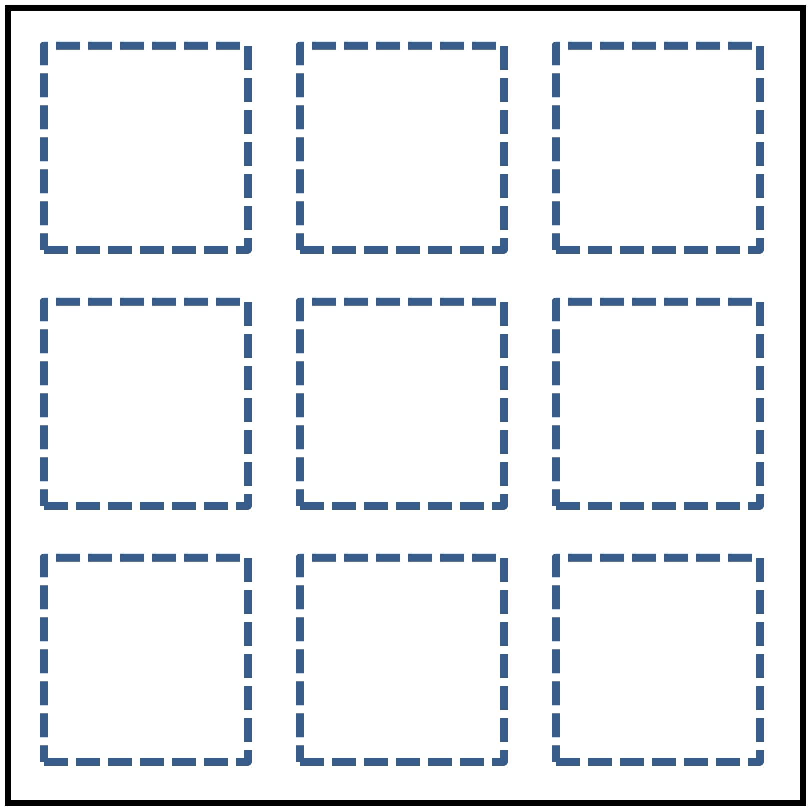
- Decompose a system in parts following the interactions



Topology of Interactions

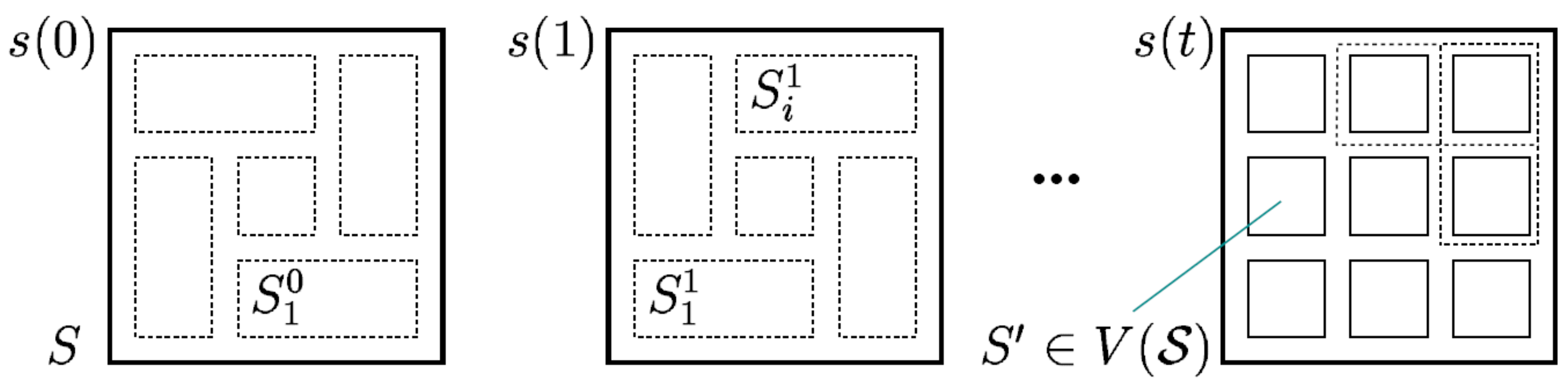
- Decompose a system in parts following the interactions

The interactions decomposes the systems into elementary parts
An interaction implies one or several elementary parts



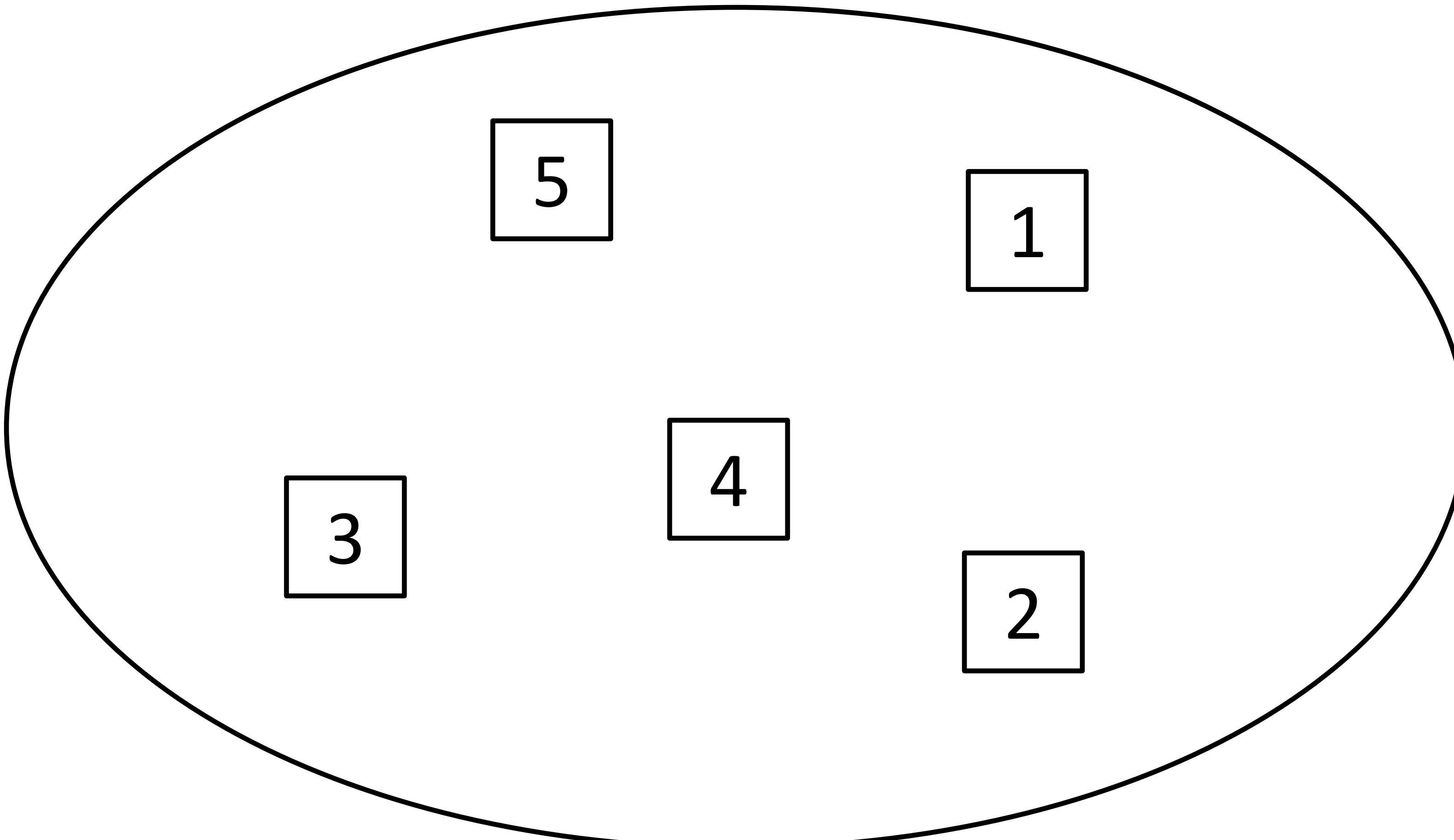
Topology of Interactions

- ***Interaction*** based modeling/simulation
  - Interactions in dynamical system
    - $s(t)$: state of the system at time t
    - S_i^t: i^{th} *sub-system where an interaction occurs* at time t
  - The successive partitions give rise to a topology on S
    - Basic elements in interaction: **points**
    - Spatial organization of the interactions: **topology of interactions**
    - Different kinds of interaction: **local evolution laws**



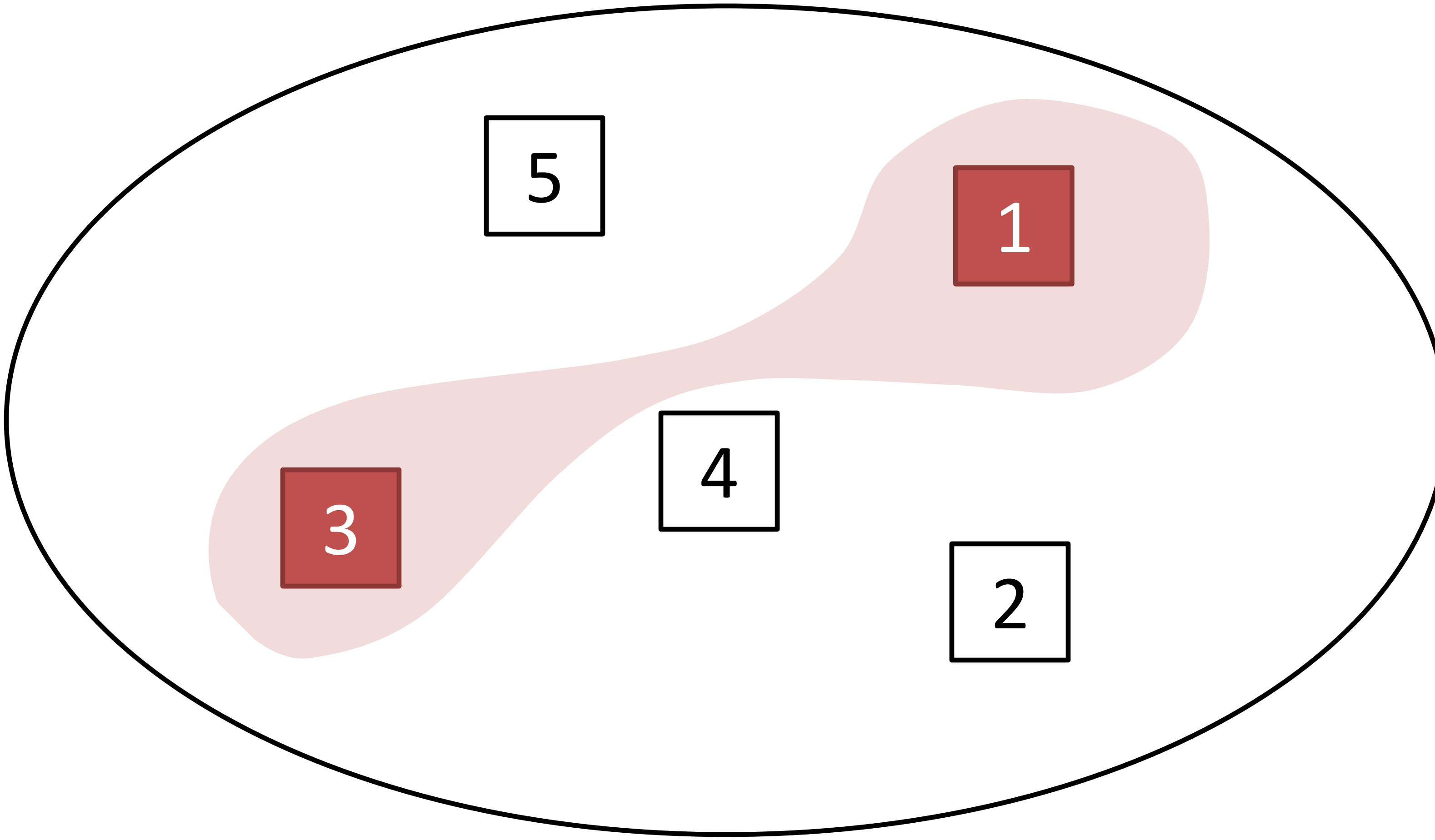
Topology of Interactions

- Initial State



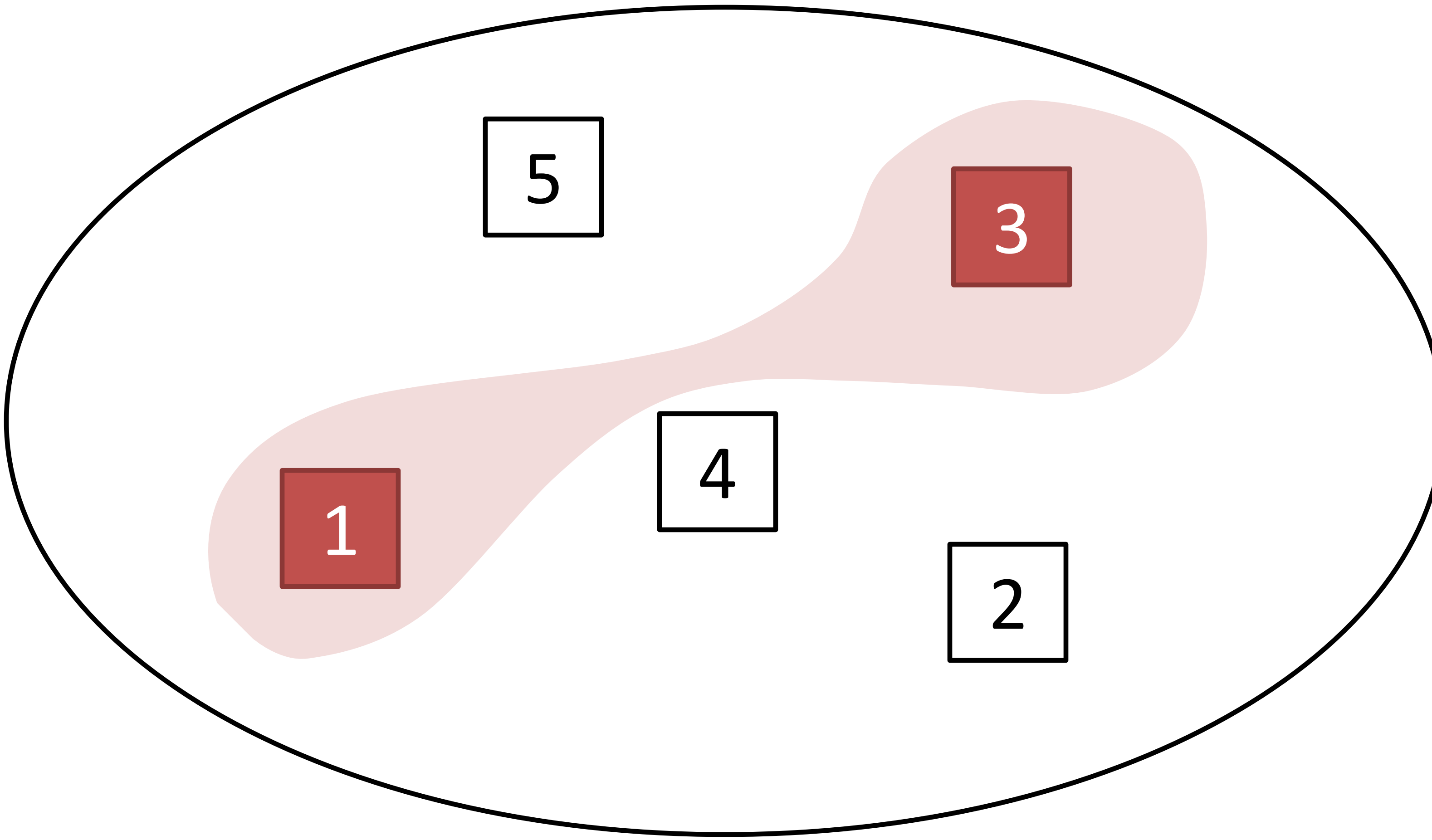
Topology of Interactions

- Interaction 1



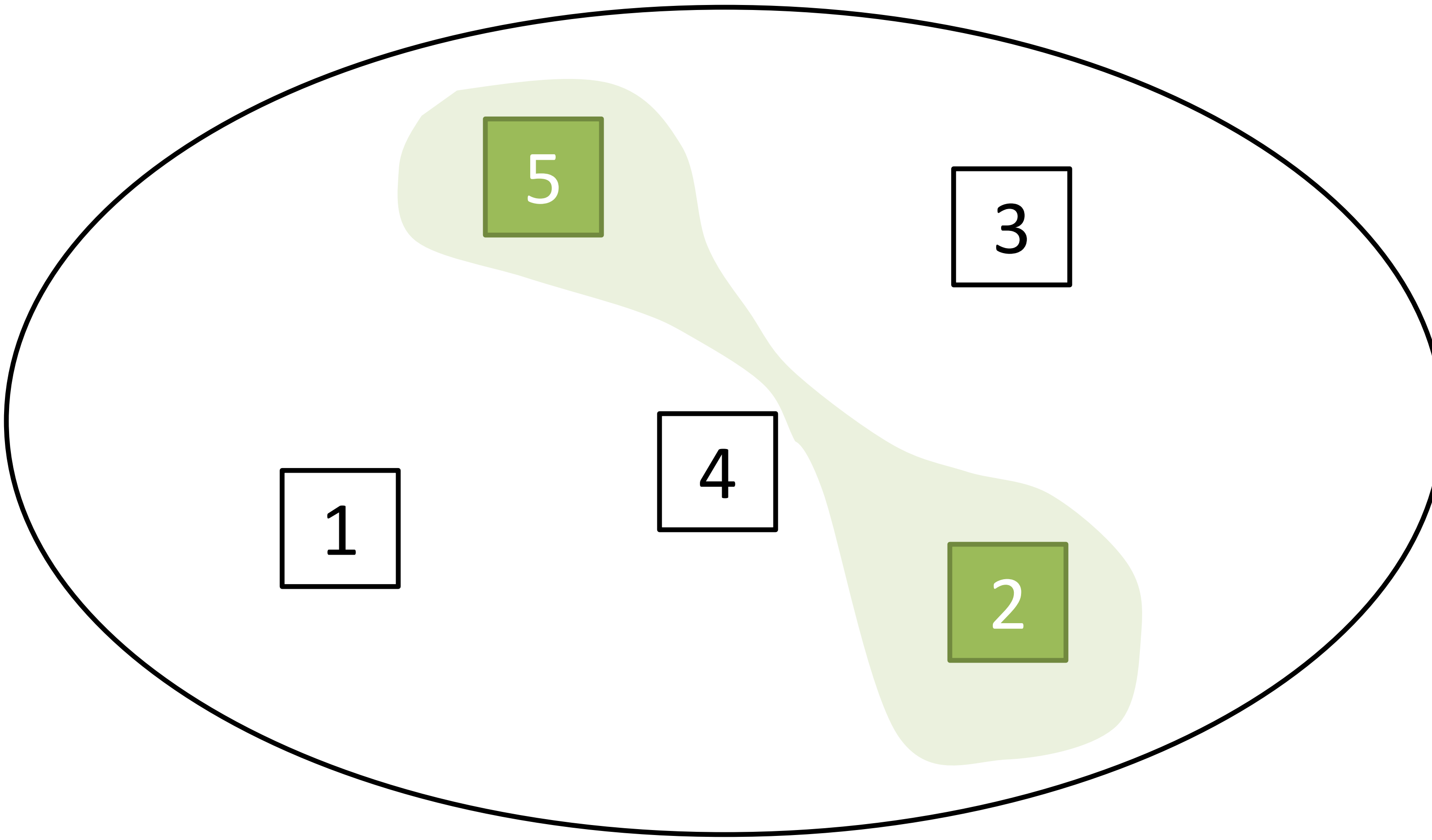
Topology of Interactions

- Interaction 1



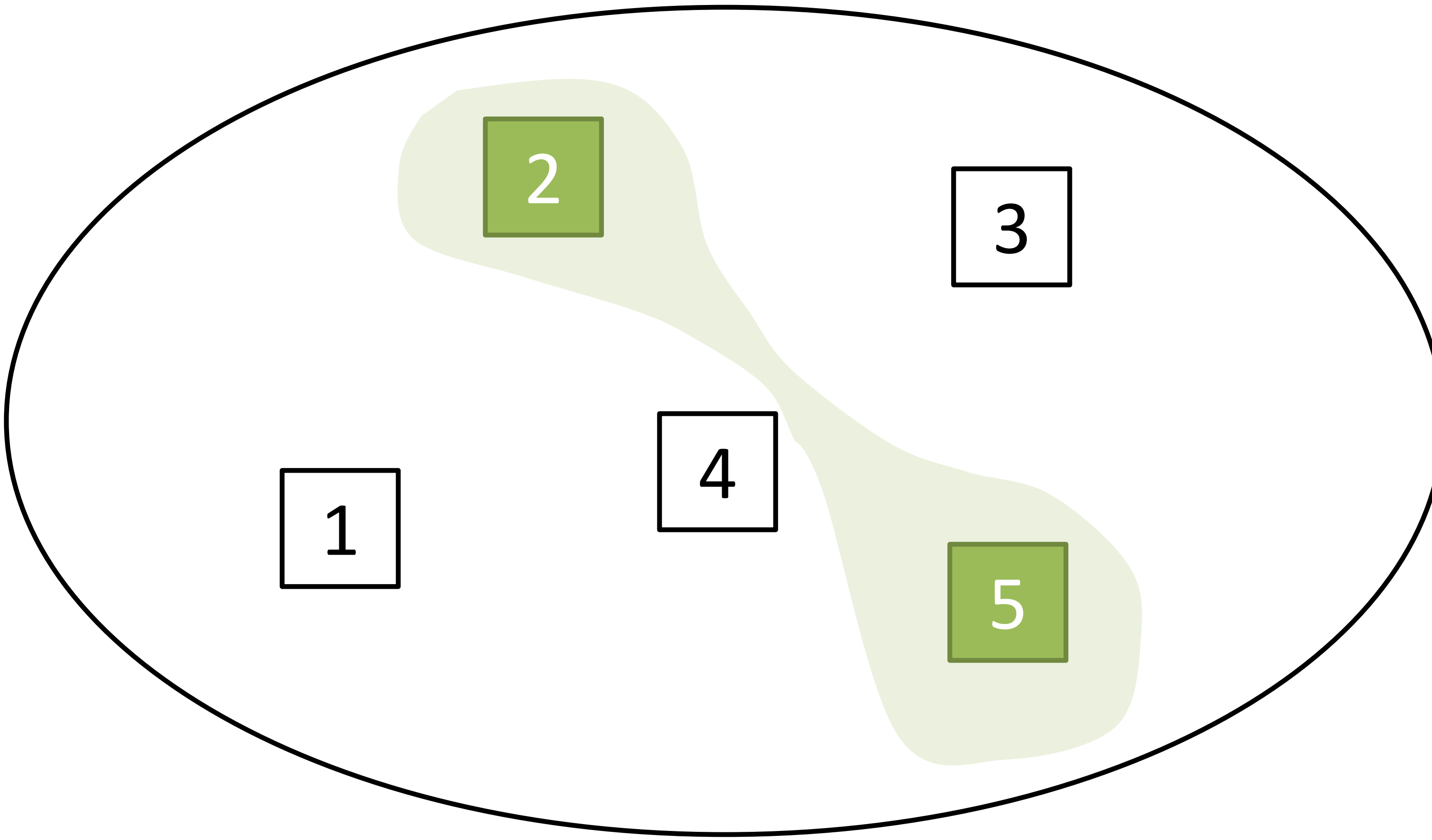
Topology of Interactions

- Interaction 2



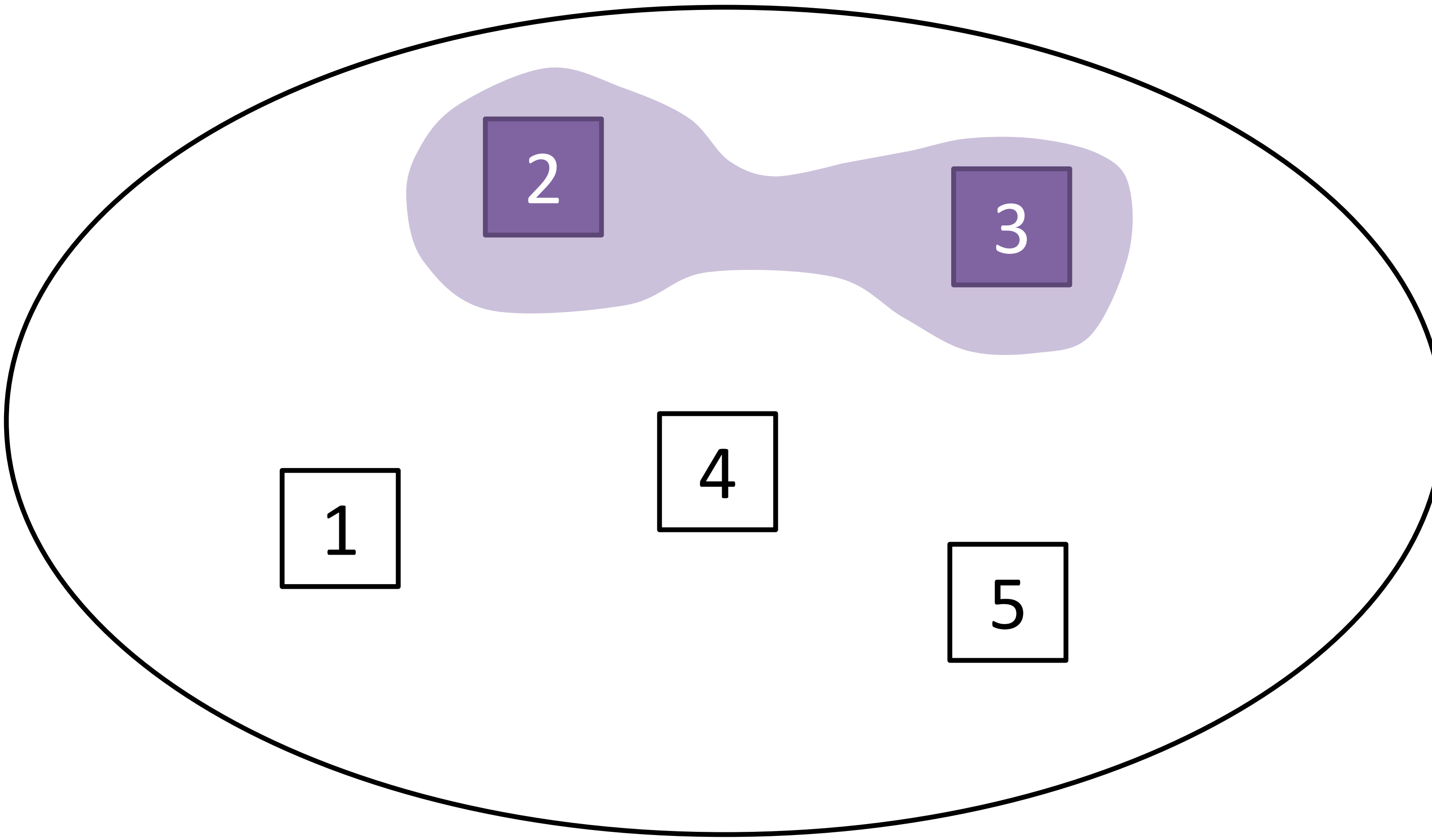
Topology of Interactions

- Interaction 2



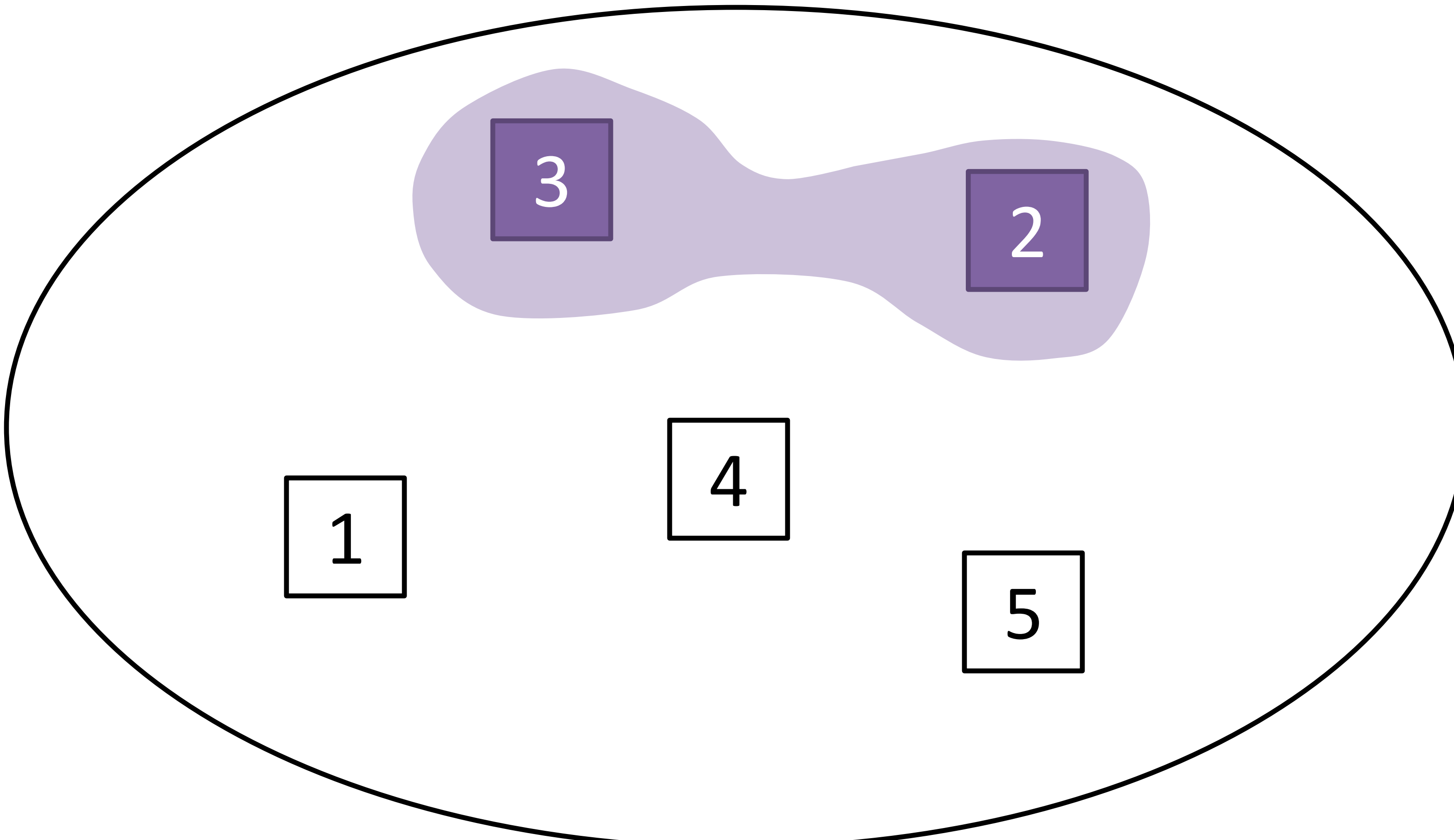
Topology of Interactions

- Interaction 3



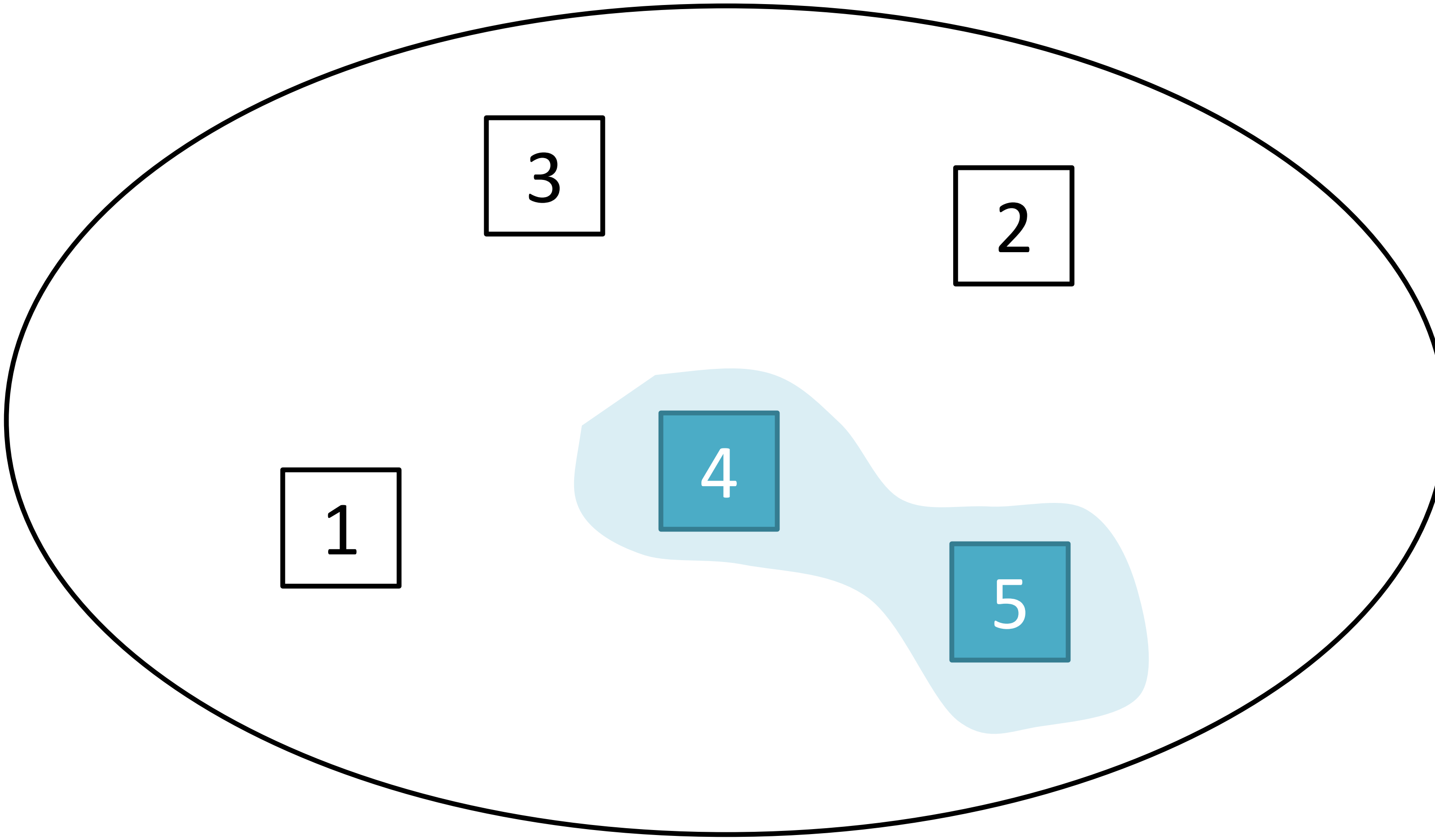
Topology of Interactions

- Interaction 3



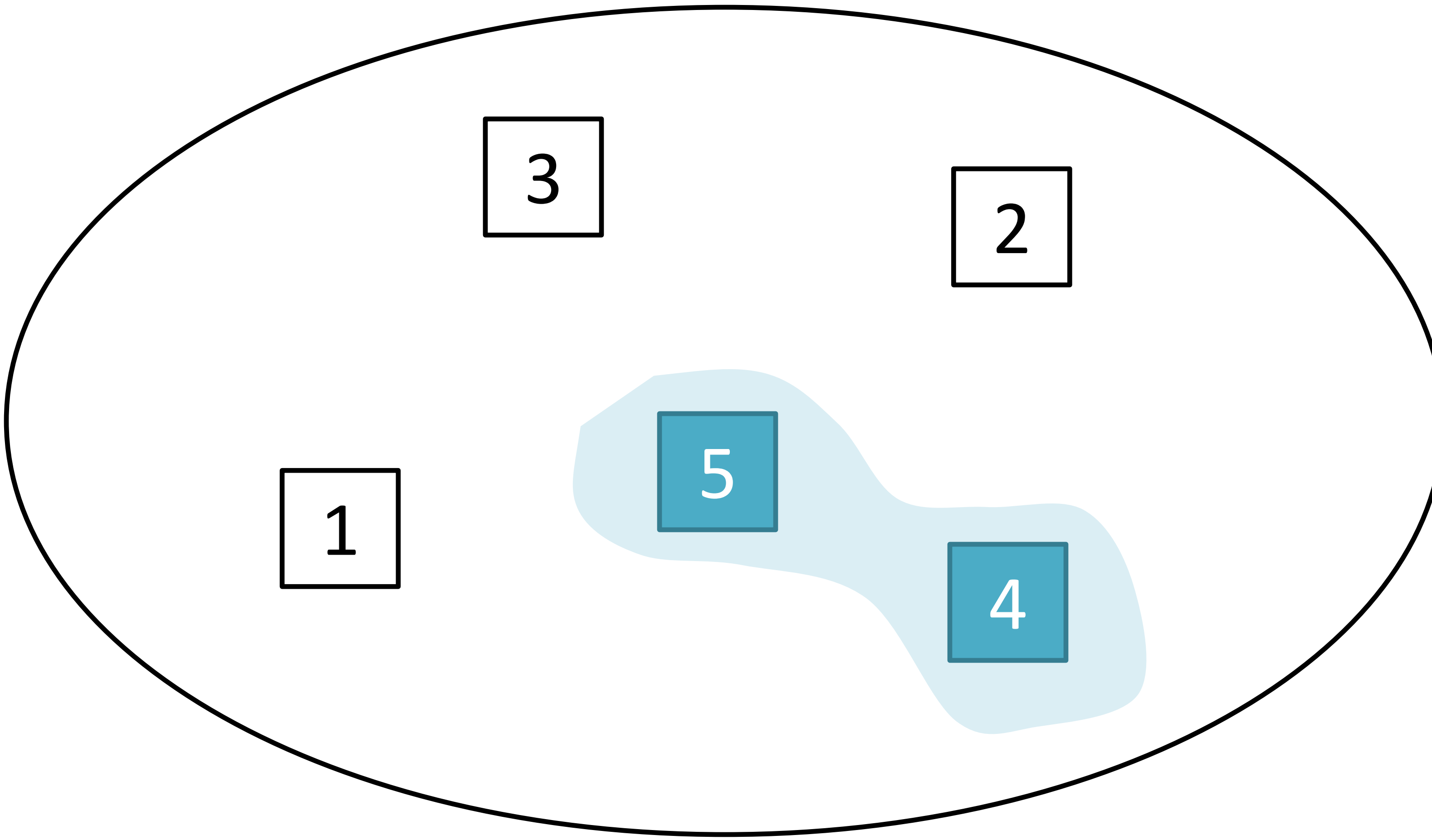
Topology of Interactions

- Interaction 4



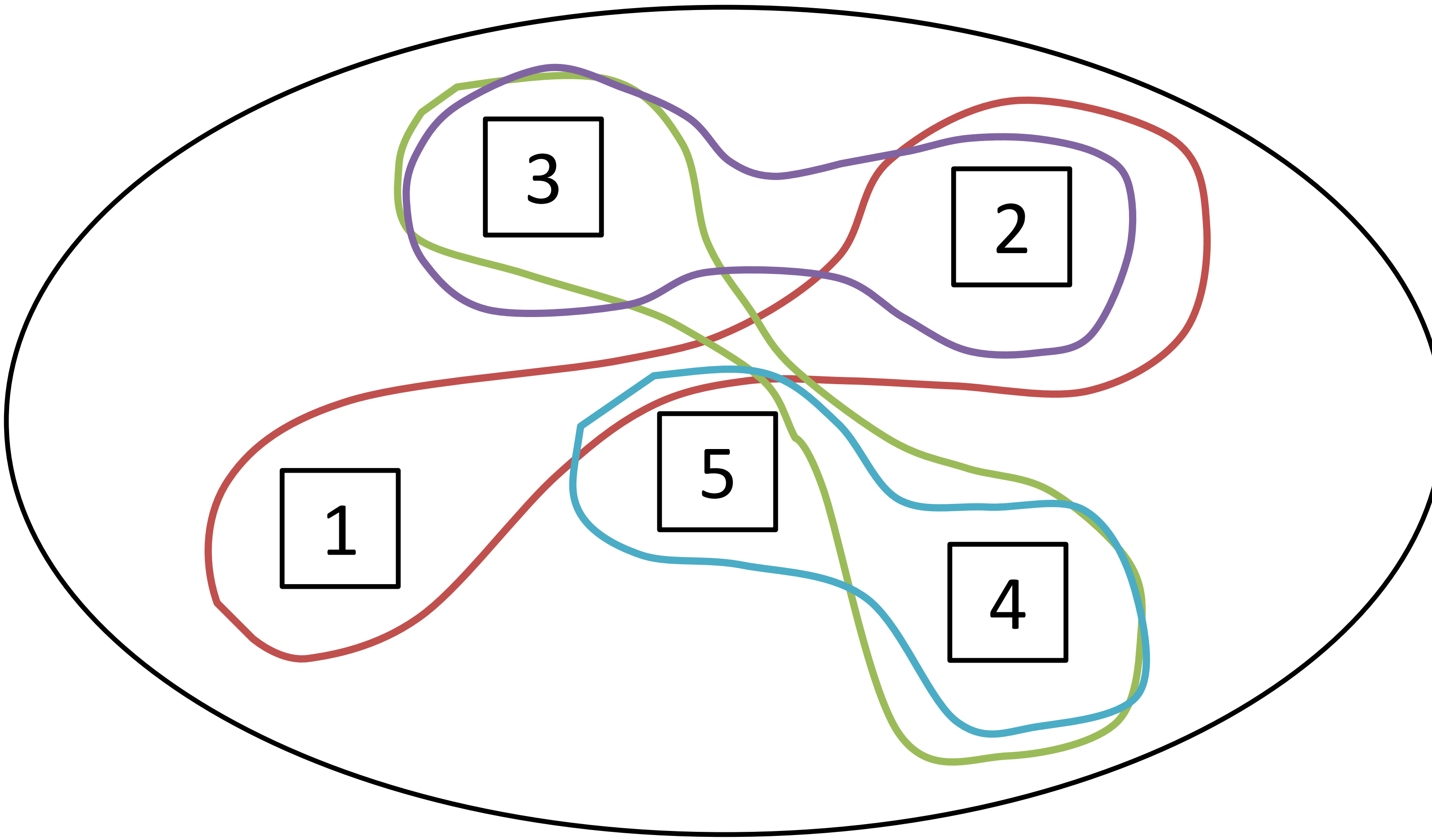
Topology of Interactions

- Interaction 4



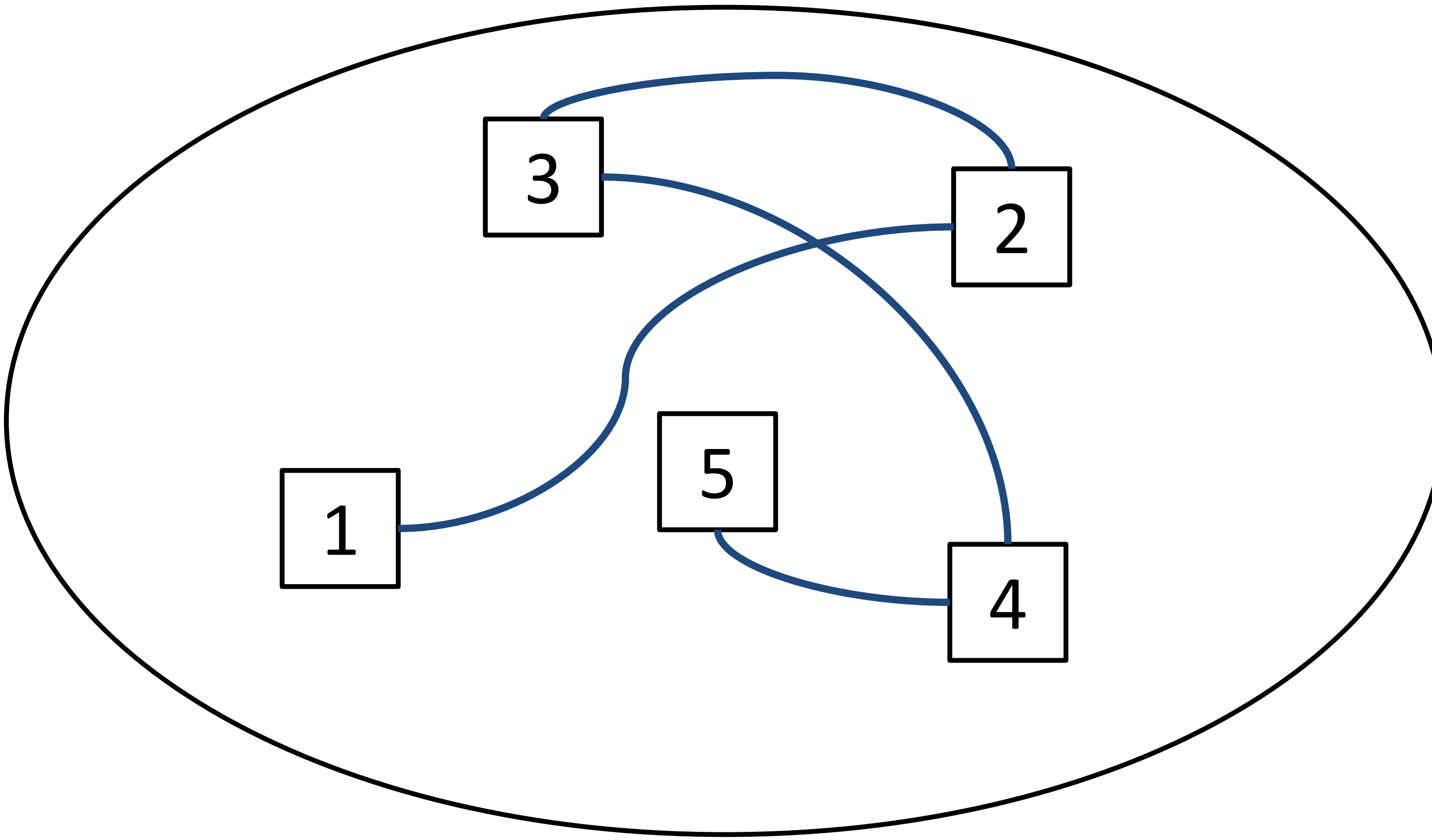
Topology of Interactions

- Partition Based on Interacting Regions



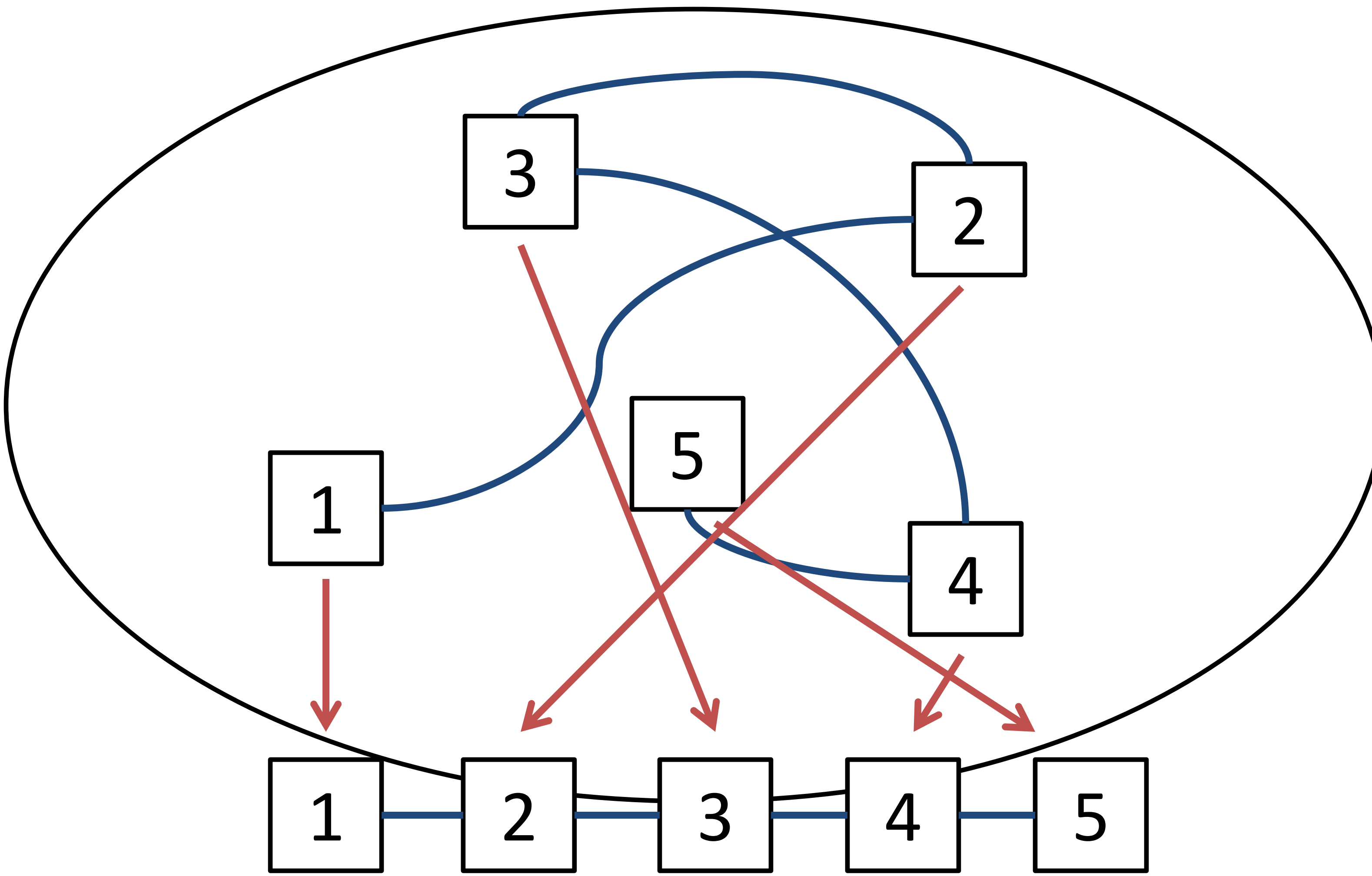
Topology of Interactions

- Induced Neighborhood Relationship



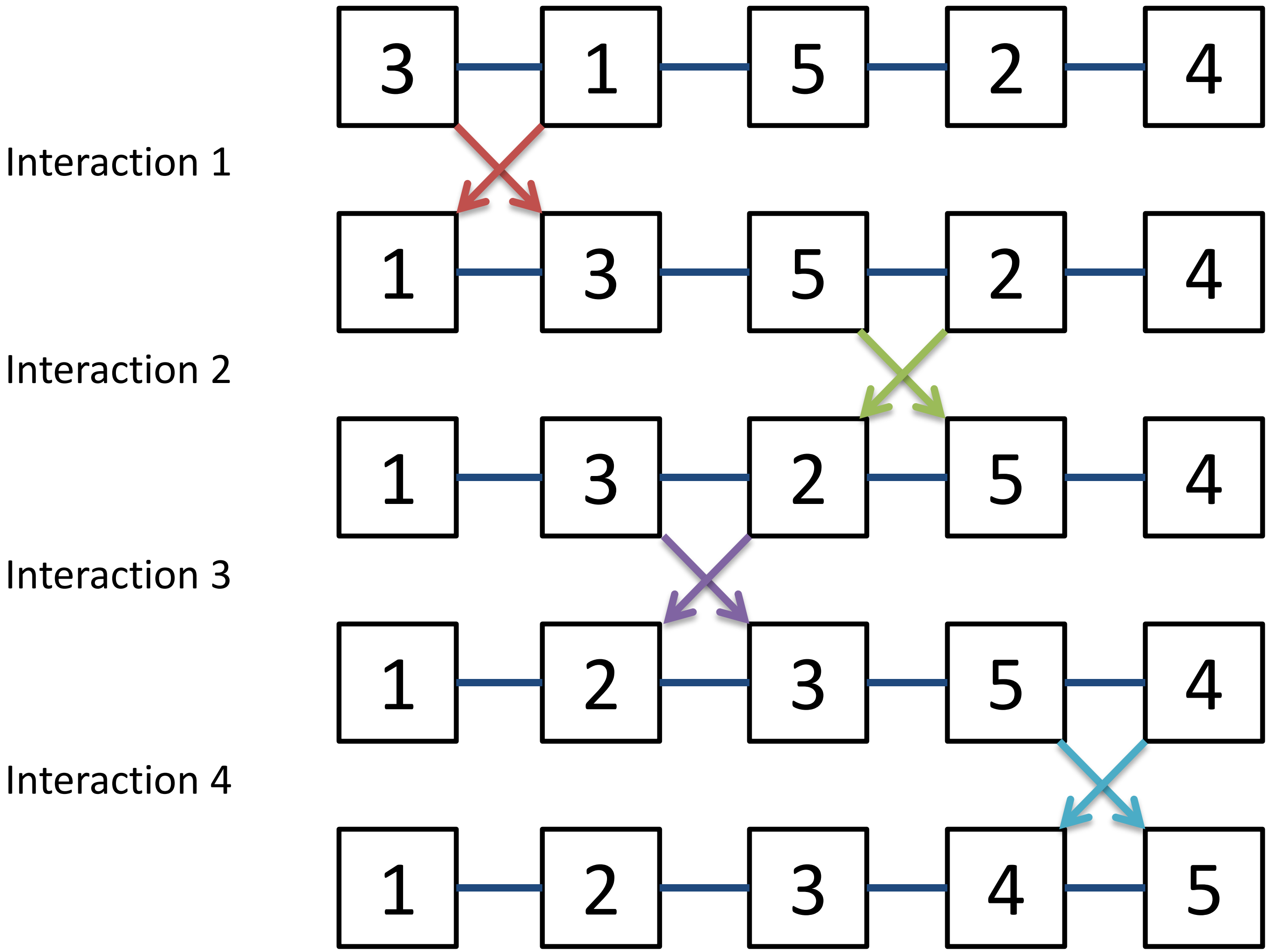
Topology of Interactions

- Topology of Sequence



Topology of Interactions

- Evolution Analysis



Topology of Interactions

- Big picture

1. Describe a dynamical system through its parts
2. Two parts are neighbors if they can potentially interact
3. Each part is characterized by a (local) state
4. The global state of the system is the “sum” of its local states and their topological organization
5. An interaction makes evolve a (small) subset of local states
6. An interaction potentially changes the topological organization of state

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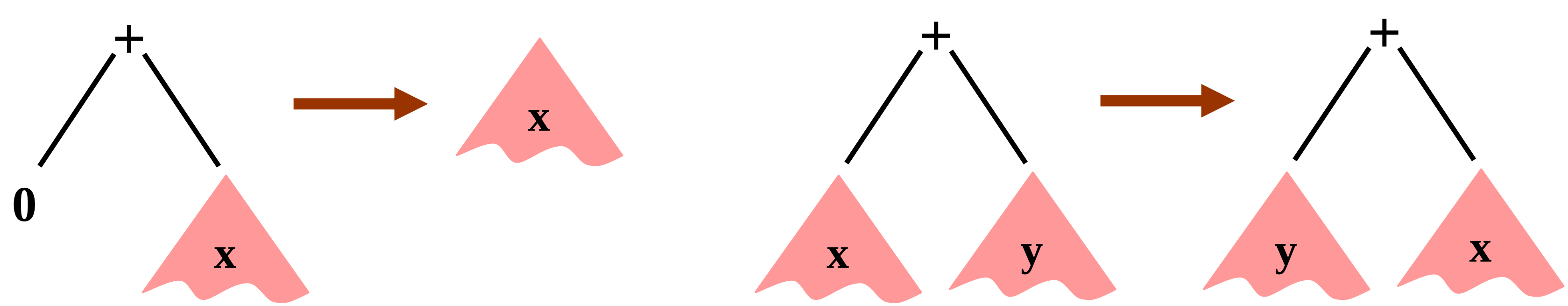
MGS: a DSL for $(DS)^2$

- Requirements
  - **Discrete**: representation of *populations of entities*
  - **Local**: the global behavior *emerges* from local interactions
  - **Declarative**: close to a *mathematical* specification

- *Rewriting* Techniques
  - Formalization of the equational reasoning

    Substitution of a sub-part of an object by another one
  - Example: simplification of arithmetical expressions

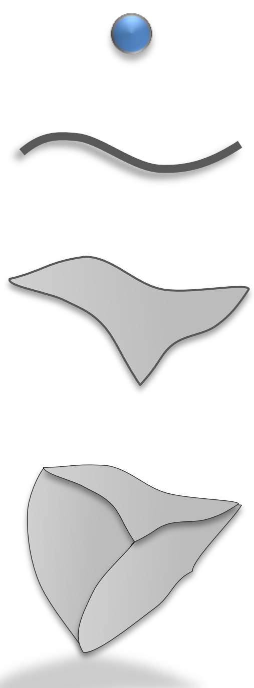


MGS: a DSL for $(DS)^2$

Dynamical Systems	Rewriting Techniques
Model	**Definition**
State (*space/topology of interactions*)	Data Structure
hierarchical organizations	formal trees (terms)
arbitrary organizations	graph, *topological collection*
Evolution Function	Rewriting System
interaction ⇒ product	$\alpha \Rightarrow \beta$ α: pattern, β: expression
local evolution laws	set of rules, *transformation*
Simulation	**Application**
Trajectories	Derivations
Time Modeling	Rule Application Strategies
discrete, event based, synchronous/asynchronous/…	maximal-parallel/sequential/ stochastic/…

MGS: Collection

- **Topological Collection**
  - Structure
    - A collection of *(topological)* ***cells***
    - An ***incidence*** *relationship (neighborhood)*

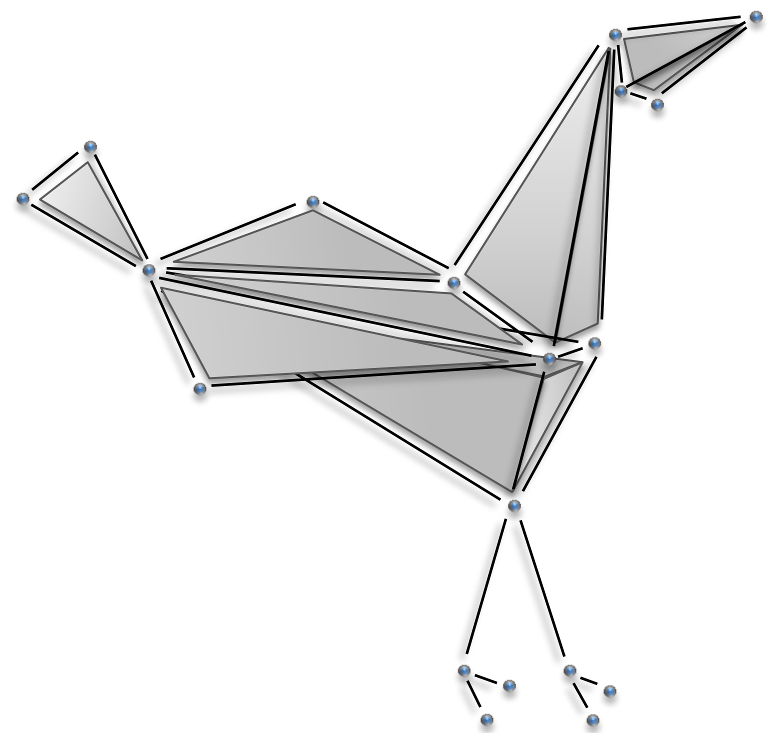


0-cell

1-cell

2-cell

3-cell

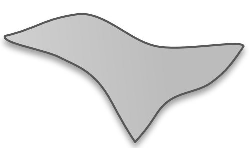


MGS: Collection

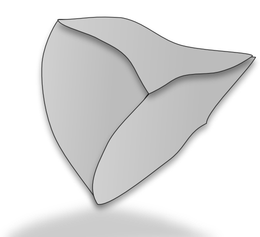
- Topological Collection
  - Structure
    - A collection of *(topological) cells*
    - An *incidence relationship (neighborhood)*
  - Data ***associated with the cells***

0-cell

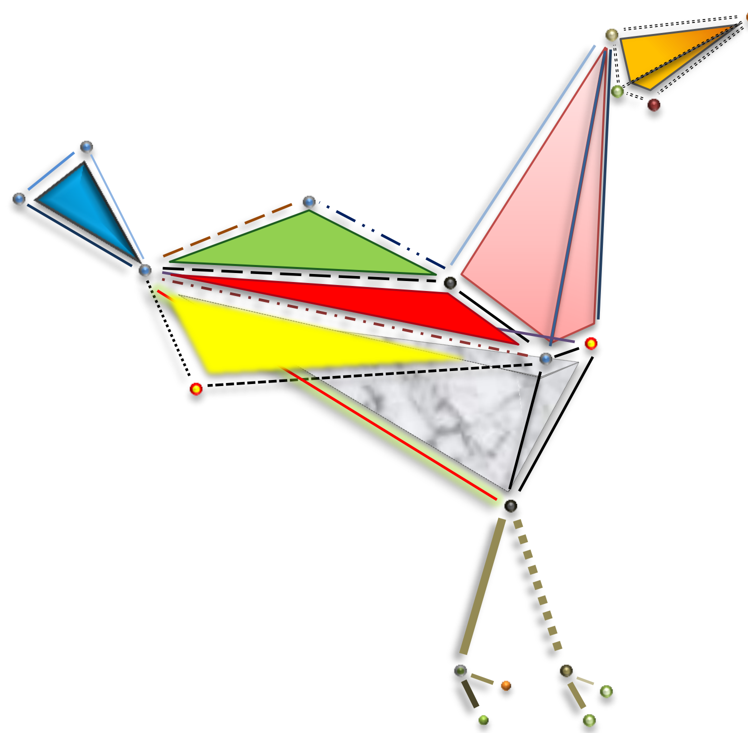
1-cell



2-cell

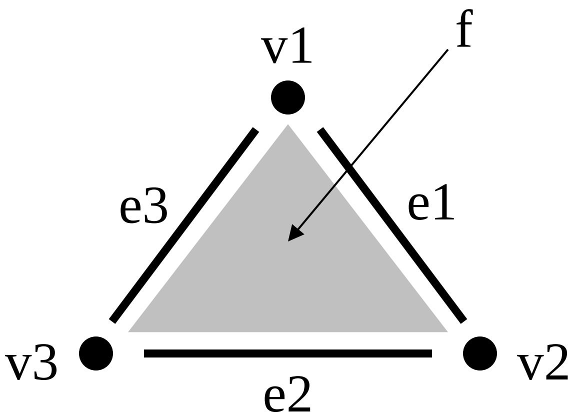


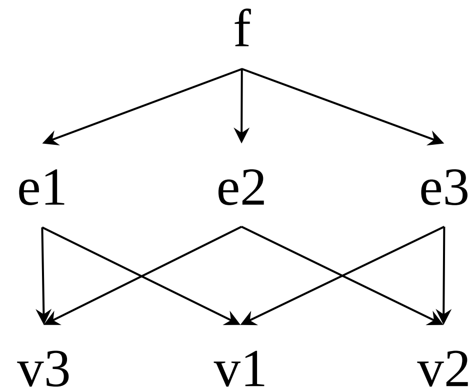
3-cell



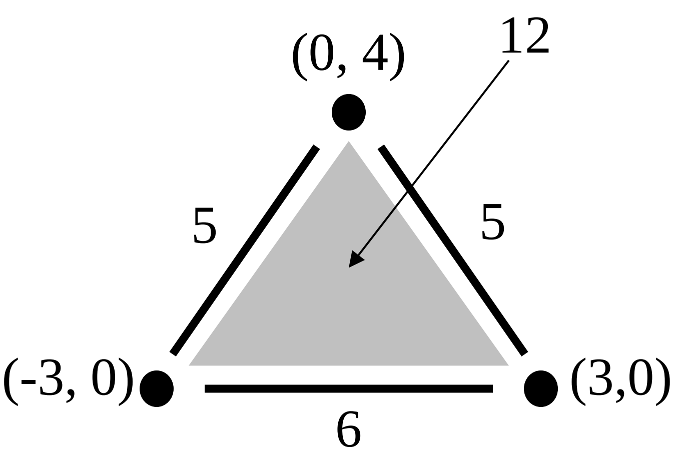
MGS: Collection

- Topological Collection
  - More formally… but still graphically





Ranked partially ordered set of cells



$$\begin{pmatrix} 0 \\ 4 \end{pmatrix}.v_1 + \begin{pmatrix} 3 \\ 0 \end{pmatrix}.v_2 + \begin{pmatrix} -3 \\ 0 \end{pmatrix}.v_3 + 5.e_1 + 6.e_2 + 5.e_3 + 12.f$$

Partial finite function labeling cells

MGS: Transformation

- Transformation
  - Functions defined by case on collections

    Each case (pattern-)*matches* a sub-collection
  - Rewriting relationship: ***topological rewriting***

trans *T* = **{**

$pattern_1 \Longrightarrow expression_1$

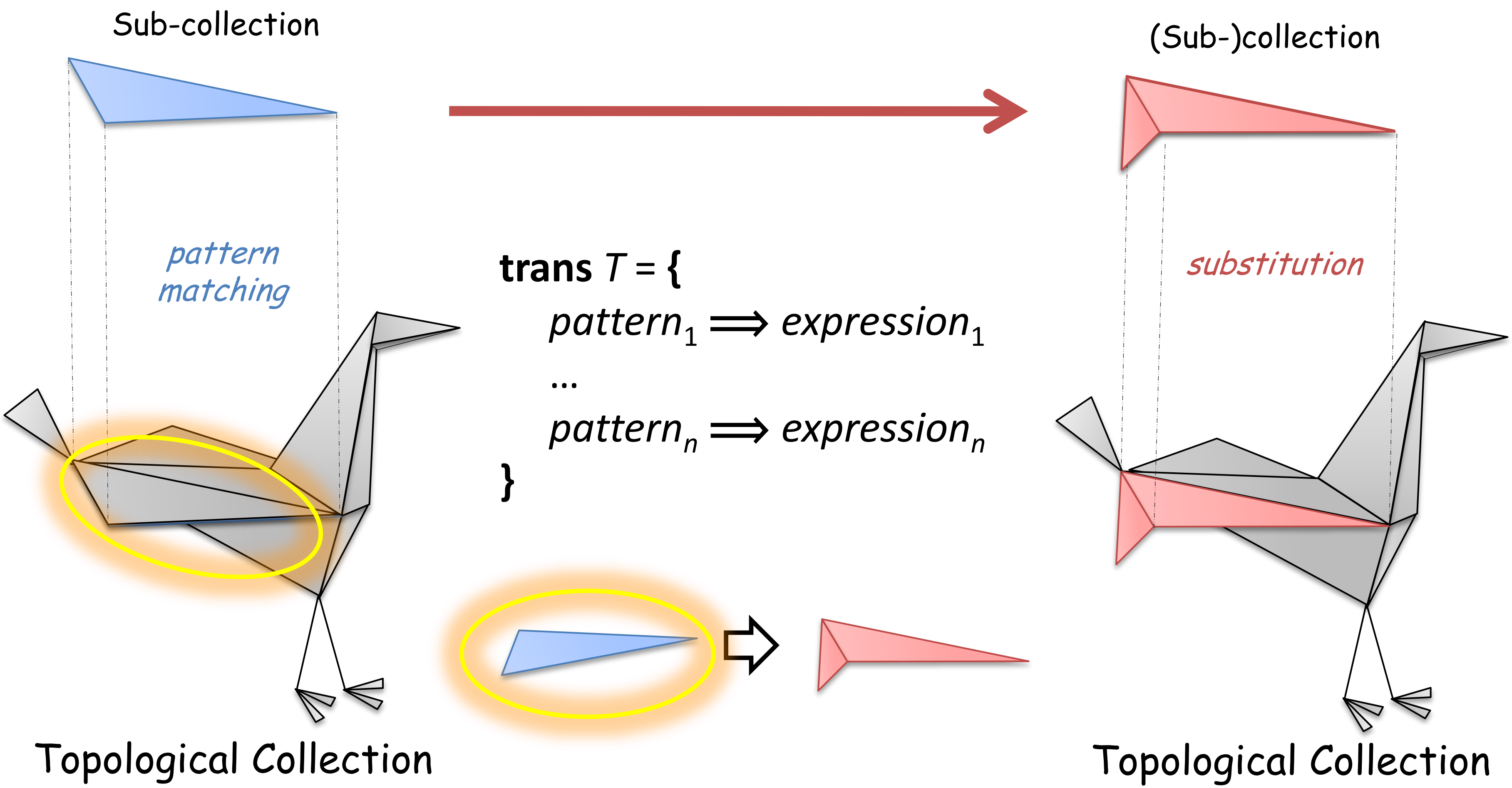
…

$pattern_n \Longrightarrow expression_n$

}

MGS: Transformation

- Transformation



MGS Examples: *dynamic on shape*

- Diffusion Limited Aggregation
  - Local evolution laws

  - MGS specification

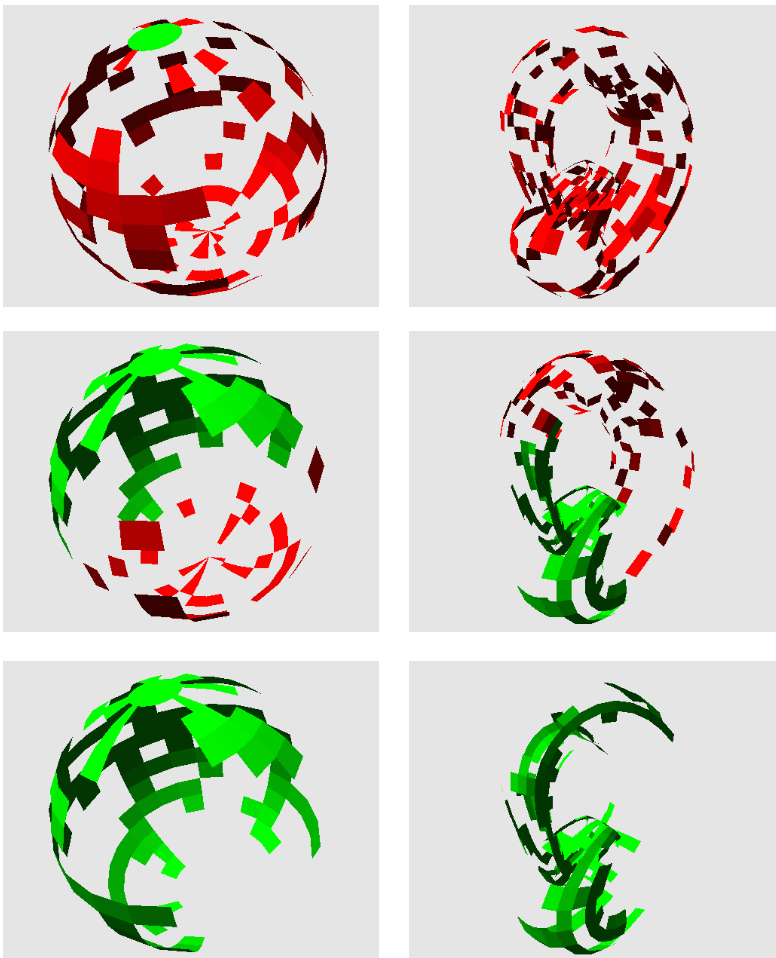
```
trans dla = {
   `red, `green   => `green, `green ;
   `red, <undef> => <undef>, `red
} ;;
```

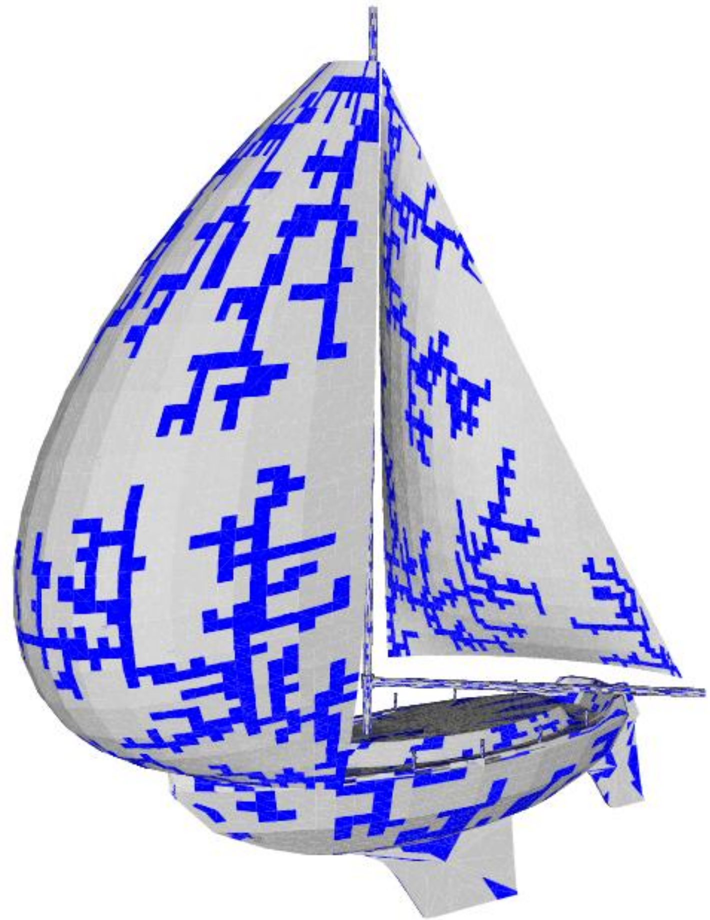
Neighborhood = interaction

Empty place

MGS Examples: *dynamic on shape*

- Diffusion Limited Aggregation

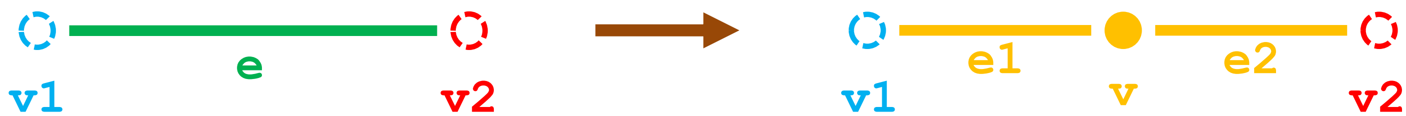




MGS Examples: *dynamic of shape*

- Topological modification

  Splitting an edge by insertion of a vertex

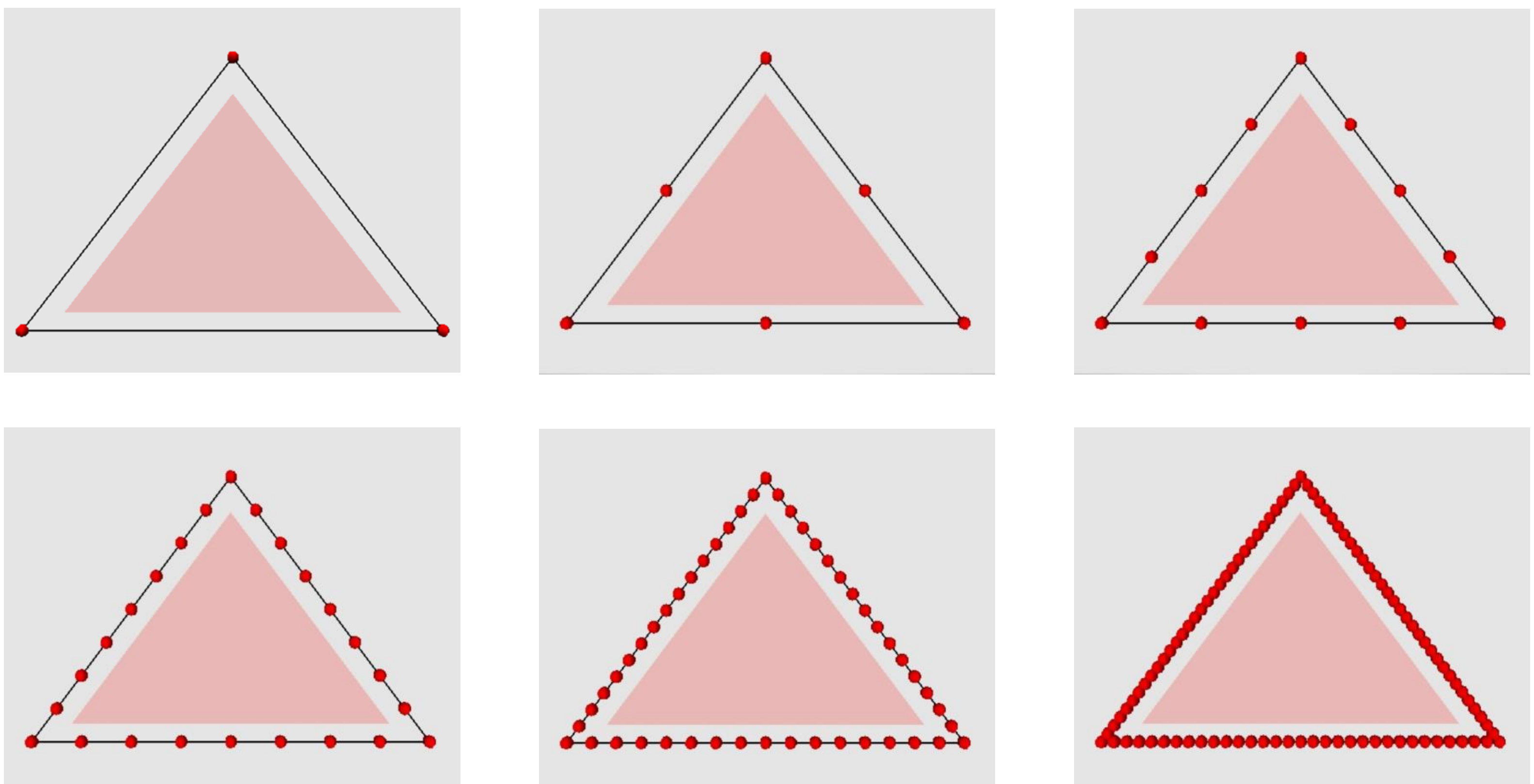



```
trans insert_vertex = {
   ~v1 < e:[ dim = 1 ] > ~v2
     =>
        letcell v  = new_cell 0 ()        ()
        and     e1 = new_cell 1 (^v1,v) (cofaces ^e)
        and     e2 = new_cell 1 (^v2,v) (cofaces ^e)
        in
          (some expression) * v
}
```

MGS Examples: *dynamic of shape*

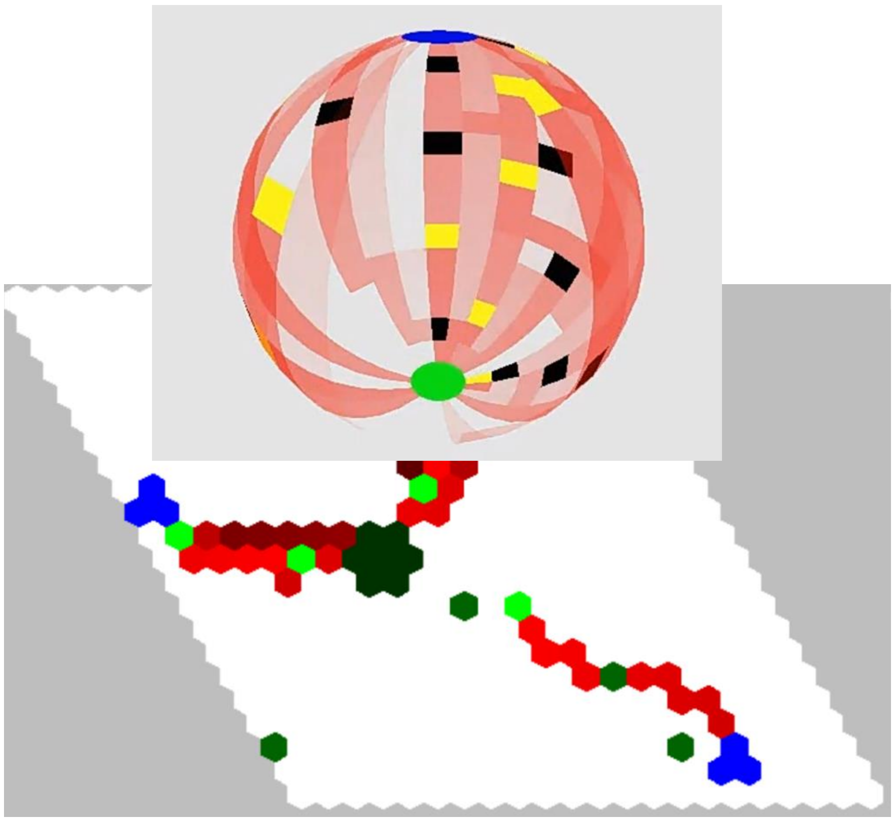
- Topological modification

  Splitting an edge by insertion of a vertex



MGS Examples

- Multi-Agents Systems



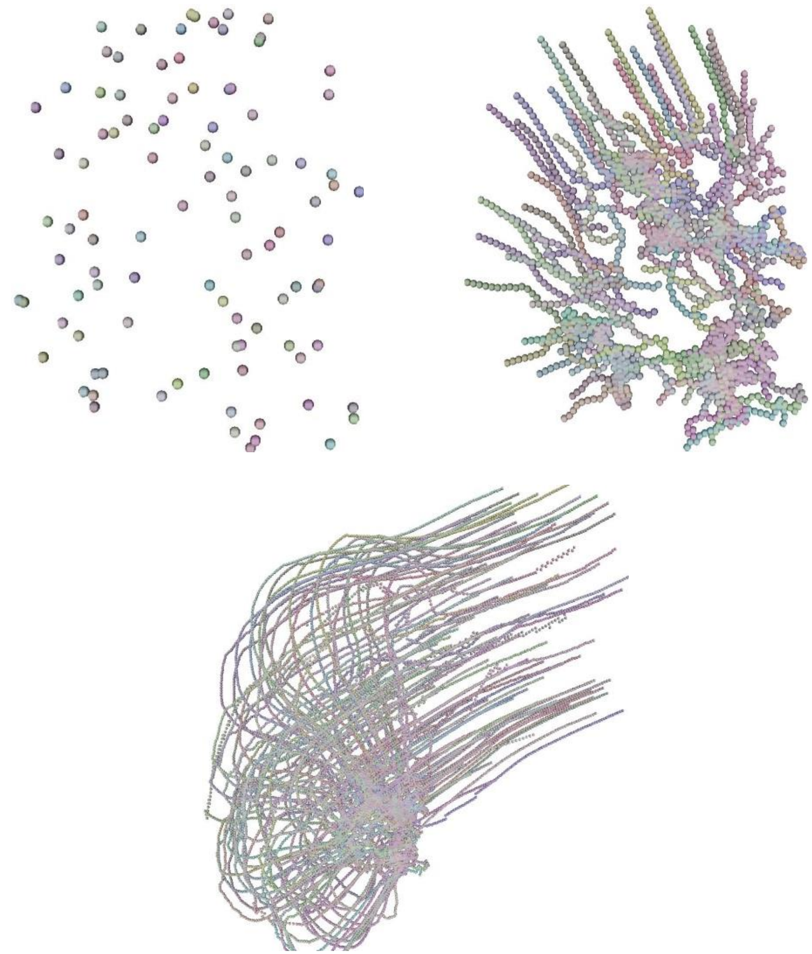
Ants foraging

One transformation, different topologies

polytypism

Boids (Reynolds, 86)

No leader, 3 evolution rules, coherent global behavior

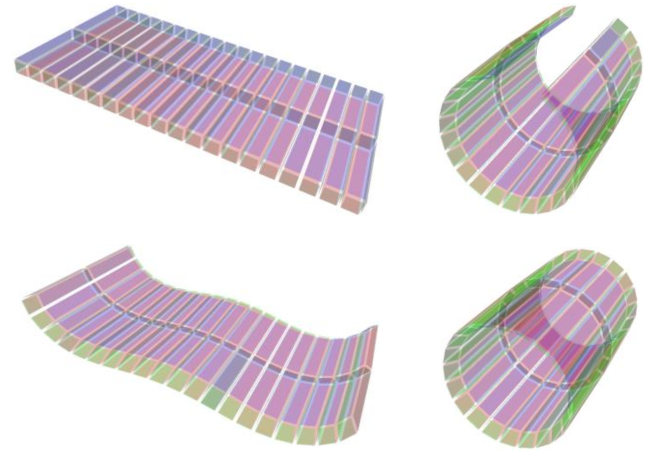


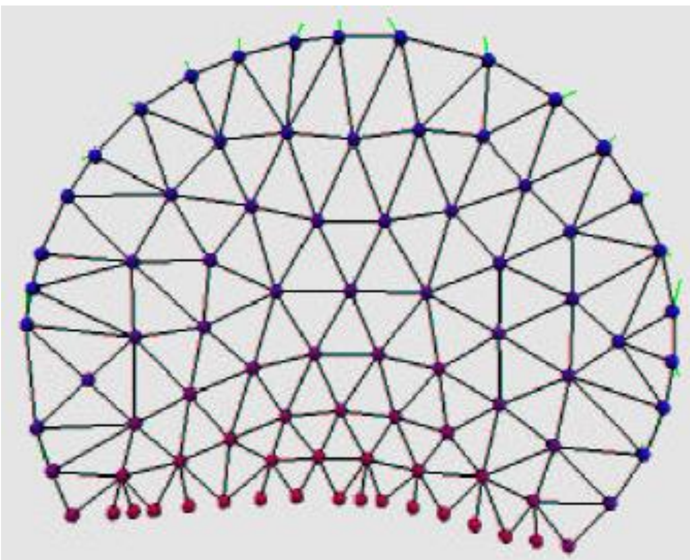
MGS Examples

- Integrative Biology

Neurulation

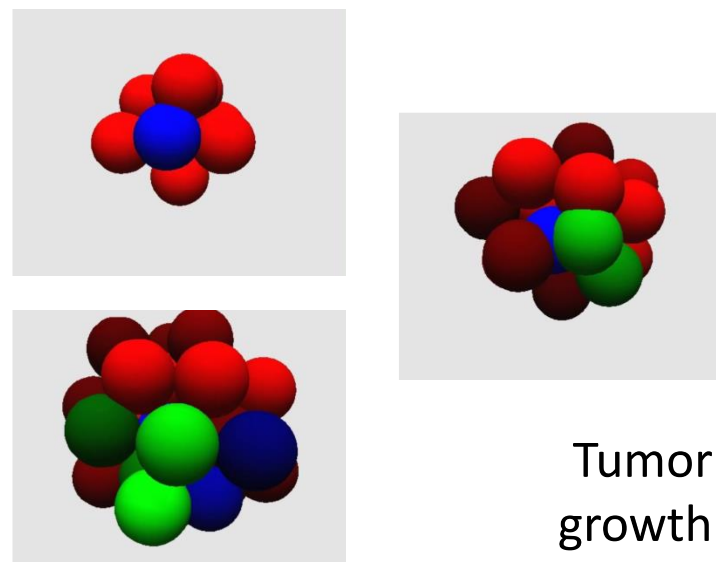
Topological surgery





Cellular motility

Adaptive mesh

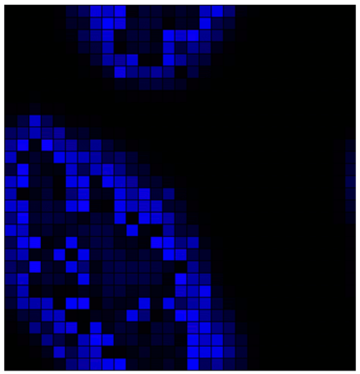


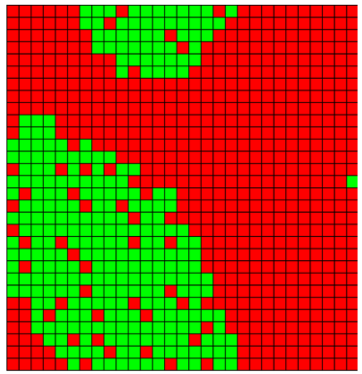
Tumor growth

MGS Examples: *synthetic biology*

Qualitative models

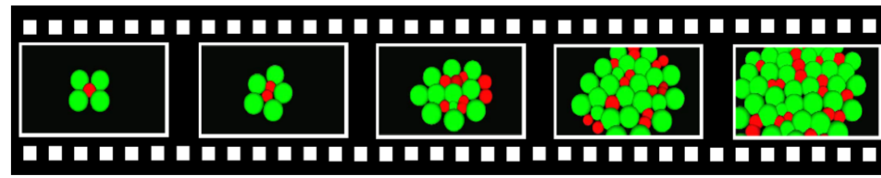
Differentiation & survival





Cellular automata

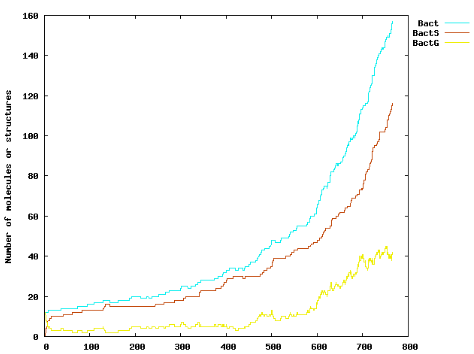
Spatial evolution of the population

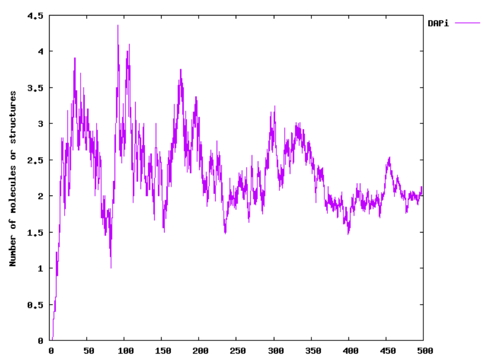


Mass/spring system

Quantitative models

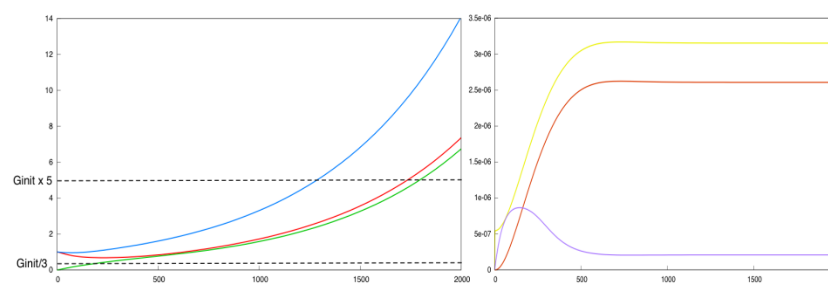
Noise sensibility





Gillespie's SSA

Robustess



Kinetics model (ODE/PDE)

Outline

- Introduction to MGS
  - Interaction-based modeling
  - Presentation of MGS
- Demonstrations
  - Lindemayer Systems
  - Chemical-like Systems
  - Cellular Automata
  - Multi-agent Systems